

# Module 4: Music & Math

Art in Mathematics (AiM)



## MUSIC & MATH

**TIME FRAME:** 12 days

**ENDURING UNDERSTANDINGS:**

Music can be modeled and refined/changed with mathematics.

**ESSENTIAL QUESTIONS:**

1. How do translations affect the sound or melody of a tune?
2. How can sine functions help us analyze sound or tones?

**TARGET AUDIENCE:**

Students needing additional real-life music applications for Algebra II, Precalculus, and Trigonometry topics with a “wow” factor that includes: function notation, translation of functions, application/translation of sine functions.

**CRMS:**

Functions 8.2b, 8.2c, 8.3a, 8.6d, and 8.6e

**AT THE END OF THE CHAPTER STUDENTS WILL KNOW AND BE ABLE TO:**

1. Write a tune on Finale Notepad.
2. Use translations to change a melody.
3. Analyze how a particular translation will affect a tune or a function.
4. Use logger pro to tune a musical instrument.

**PRE-REQUISITE KNOWLEDGE/SKILLS:**

Graphing

**PRE-ASSESSMENT ACTIVITY:**

None

**ACTIVITIES:**

Introductory problem, **Twinkle Twinkle  
Tubular Glockenspiel  
Making Music with Wine Glasses**

**POST-ASSESSMENTS:**

Project: Write a melody and play it on wind chimes.  
Cornell Notes on Transformations of Sine Curves  
Portfolio of Work (including above mentioned notes)

**LITERACY STRATEGIES:**

Group Work  
Cornell Notes

**RESOURCES:**

**YouTube Videos at:**

<http://www.youtube.com/watch?v=VEiEBEadZFI>

<http://www.youtube.com/watch?v=rJSu12sWPFY>

**Physics Applet at:**

[http://www-personal.umich.edu/~yxl/486/ggb/sine\\_curve\\_transformations.html](http://www-personal.umich.edu/~yxl/486/ggb/sine_curve_transformations.html)

**Finale Notepad Software Program at:**

<http://www.finalemusic.com/notepad/>

**Music files at:**

[http://www.instruction.greenriver.edu/projecttime/Music/Music\\_home.htm](http://www.instruction.greenriver.edu/projecttime/Music/Music_home.htm)

## DAILY PLAN

### Day 1:

- Introductory activity: **Twinkle, Twinkle.**
- **Graphing Calculator Activity** – review how to enter lists of data on calculator, enter Twinkle, Twinkle notes into graphing calculator.
- Concept Map Review (Teachers might hand out Final Project Requirements at this time also).
- Finale Notepad if time.

**HW:** Work on melody at home if possible, otherwise no homework.

### Day 2:

- Introduce function notations with Entry task of: If  $y = 2x + 1$ , what is  $y$  if  $x = 3$ , what is  $y$  if  $x = 6$ . How does this relate to  $f(x) = 2x + 1$  and  $f(3)$  and  $f(6)$ ? Students should be familiar with this notation.
- Work with students to complete Worksheet 1: Functions. Students use their graphing calculators if they want to help complete the worksheet. Make sure students get a good start on this worksheet.
- Work on melody and Finale Notepad at least 20 minutes (start work on **My Personal Song**).

**HW:** Continue work on melody. Finish **Worksheet 1**.

### Days 3- 4:

- Go over Worksheet 1. Have students work in pairs to correct work. Send students to board to answer any questions that might come up about the worksheet. Students enter the tables from worksheet on graphing calculator to compare graphs.
- Students take Cornell Notes on Function Translations.
- 20 minutes on Finale Notepad Day 3
- Hour on Finale Notepad Day 4, correct homework

**HW: Worksheet 2: Transformations of Twinkle, Twinkle**

### Day 5:

- Input information from worksheets into Finale Notepad ( $f(x - 4)$ ,  $f(x) + 2$ ) Divide class into teams. Assign different transformations to different teams.
- Students finish Individual Tune and write table of notes.
- Share out tunes.

**HW:** Students work on translating their melodies, Start **Worksheet 3, My Personal Song**.

**Day 6:**

- Watch YouTube Clips at:  
<http://www.youtube.com/watch?v=VEiEBEadZFI>  
<http://www.youtube.com/watch?v=rJSu12sWPFY>
- **Worksheet 4: Sine Curve Activity** (Physics Applet on Sound on computer) at:  
[http://www-personal.umich.edu/~yxl/486/ggb/sine\\_curve\\_transformations.html](http://www-personal.umich.edu/~yxl/486/ggb/sine_curve_transformations.html)
- Students take Cornell Notes on their own about effects of a, b, c, and d on sine waves. Students use Sine Waves Cornell Notes Worksheet.

**HW:** Continue work on **Worksheet 3: My Personal Song**.

**Day 7:**

- **Worksheet 5, Tuning Forks Activity** using LoggerPro.
- HW:** Continue **Worksheet 5**.

**Days 8-10:**

- Groups tune wind chimes, wine glasses, or other objects.
- Activity: **Tubular Glockenspiel**
- Activity: **Making Music with Wine Glasses**
- Each group tunes objects for one note, and then whole class can use tuned instruments to play their Personal Song.

**Days 11-12:**

- Groups play one of melodies on wind chimes or other class instruments.
- Project: **Personal Tune and Translations**.
- Students Turn in Portfolio, **Music Unit: Final Project/Portfolio**

## Music Unit: Introductory Activity

### Twinkle, Twinkle

After Finale Notepad has been downloaded on the computer, play Twinkle, Twinkle. Handout Finale Notepad sheet with notes. Have one of students explain the notes. Discuss how we could think of the notes of Twinkle, Twinkle as ordered pairs of a function. The x coordinates of the points is the number of the note/quarter time (i.e. first note, second note, etc) In Twinkle, Twinkle a half note will count as two beats, so it will have two consecutive x coordinates and the same y coordinate. Look at the first sheet of Worksheet 2: Twinkle, Twinkle Translations.

Ask students how could a person change Twinkle, Twinkle music, but still have it recognizable as Twinkle, Twinkle? How could a singer change it? Look for answers such as sing it in a different pace, octave, rap or jazz. Talk about that if we were to think of it at a different pace or higher octave, what we are doing is really a transformation of the function (stretching along the x axis or y axis, or translating along the x or y axis). Play Twinkle, Twinkle at different tempos (along the top of the menu of Finale Notepad there is a spot to change the tempo from quarter note to half note or eighth note). Play Twinkle, Twinkle  $y + 2$ . Note that it doesn't sound quite right; there are problems with sharps and flats if you just lift all the notes two spaces up, but the idea is the same.

With graphing calculators have students input x values into L1 and y or  $f(x)$  values into L2 and then set up scatterplot and graph. Have them compare the graph on the calculator with the music of Twinkle, Twinkle. They should note that the general shape is the same. Take the time to show students how to make a scatterplot on the graphing calculator if they have not done so before. Pair students up, so there is an experienced graphing calculator person on each team. This is a valuable lesson for upcoming worksheets.

Hand out the concept map for the unit. Explain that in this unit students will look at music through the eyes of a mathematician. We will be studying music as a transformation of functions. We will also look at sound as a sine function. Students will write their own piece of music, transform it, and hopefully have time to play it on some homemade musical instrument (wind chimes, wine glasses, etc.)

What time there is left, have students play around with Finale Notepad. Students can work in pairs if they are uncomfortable with music software/knowledge.



LAST UNIT / Experience	CURRENT UNIT Music Unit (AiM Unit)	NEXT UNIT / Experience
<p><b>Unit / Homework Schedule</b></p> <ul style="list-style-type: none"> <li><input type="checkbox"/> Calculator Activity 1</li> <li><input type="checkbox"/> Use Finale Notepad</li> <li><input type="checkbox"/> Write music</li> <li><input type="checkbox"/> Worksheet 1: Functions</li> <li><input type="checkbox"/> Worksheet 2: Twinkle, Twinkle Transformations</li> <li><input type="checkbox"/> Cornell notes on Transformations</li> <li><input type="checkbox"/> Worksheet 3</li> <li><input type="checkbox"/> Worksheet 4</li> <li><input type="checkbox"/> Cornell notes on Sine Curves</li> <li><input type="checkbox"/> Worksheet 5</li> <li><input type="checkbox"/> Portfolio</li> <li><input type="checkbox"/></li> </ul>	<div style="text-align: center;"> <p>Studying music through the eyes of a mathematician.</p> </div>	<p><b>UNIT VOCABULARY</b></p>
<p><b>UNIT ESSENTIAL QUESTIONS</b></p> <ol style="list-style-type: none"> <li>1. How do mathematical transformations (stretching, shrinking, translations) affect the sound of melody of a tune?</li> <li>2. How can sine functions help us analyze sound or tones?</li> </ol>		

## **MUSIC UNIT: FINAL PROJECT/ASSESSMENT**

### **MY PERSONAL SONG**

**Student Name:**

**Group Members:**

#### **Portfolio/Presentation**

At the end of the music unit students will turn in a Portfolio of their Work. The portfolio will include:

1. Finale Notepad Copy of their Personal Song.
2. Worksheet 3, Translations of their Personal Song.
3. Cornell Notes on Transformations of the Sine curve.
4. Written paragraphs answering the guiding questions for the unit.
5. Homework.

By the end of the unit students will have performed or played:

1. Their song for the teacher or class.
2. The note they tuned on a wind chime or wine glass or pipe.
3. One transformation of their song for the teacher or class.
4. Each group of four students will play one tune on the tuned instruments.

#### **Rubric for Project: Portfolio/Presentation**

- 30% of the points will come from the copy of their Personal Song on Finale Notepad and the playing of the tune for the teacher or class. The tune must be at least 40 notes long.
- 20% of points for Worksheet 3 (4% points for each T chart)
- 10% for Cornell Notes
- 10% for Guiding Questions
- 20% for tuning one “instrument” note
- 5% Group work on playing tune
- 5% Homework during Unit

I believe my grade should be a \_\_\_\_\_.

The reasons I believe this are:

**Students will turn in their portfolio at the end of the unit. It should include this sheet and students should indicate and justify on this paper what they believe their grade should be.**

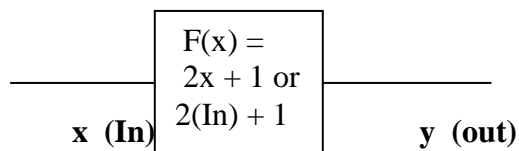
## WORKSHEET 1: FUNCTIONS

### ANSWER KEY

Fill in the T-charts and give the domain and range of each function and/or transformation.

$$f(x) = 2x + 1$$

In	Out
x	y
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	21

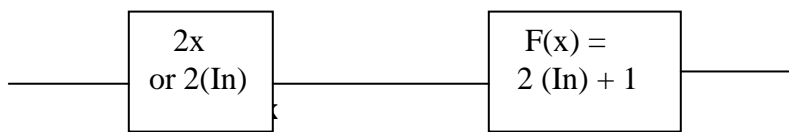


*Domain is set of numbers 1 through 10, range is set of odds 3-21*

The set of IN numbers is the domain, the set of Out numbers is the range.

$$y = f(2x)$$

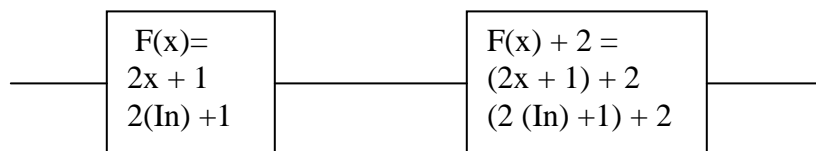
In	2x	Out
x		F(2x) = y
1	2	5
2	4	9
3	6	13
4	8	17
...		
10	20	41



*Domain is set of numbers 1-10, Range is every fourth number 5-41*

$$Y = f(x) + 2$$

x	f(x)	y = f(x) + 2
1	3	5
2	5	7
3	7	9
...		
10	21	23



*Domain is set 1-10, range is set of odds from 5 to 23*

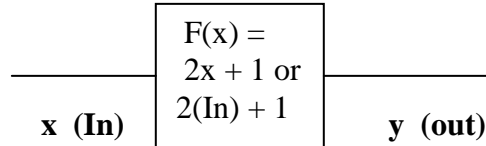


## WORKSHEET 1: FUNCTIONS

Fill in the T-charts and give the domain and range of each function and/or transformation.

$$f(x) = 2x + 1$$

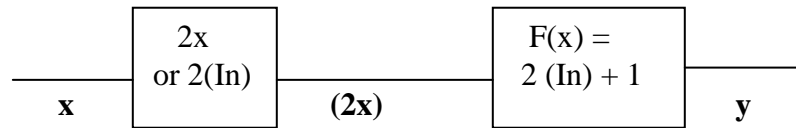
In	Out
x	y
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



The set of IN numbers is the domain, the set of Out numbers is the range.

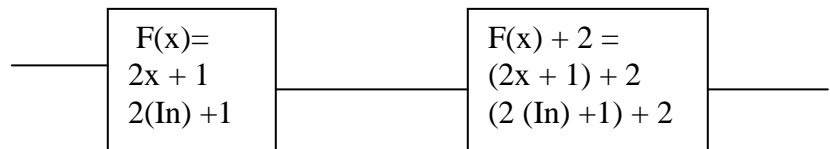
$$y = f(2x)$$

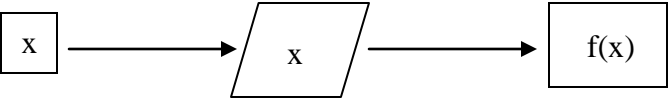
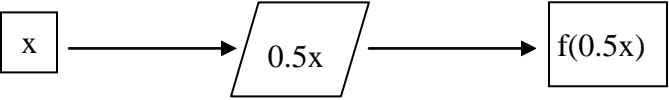
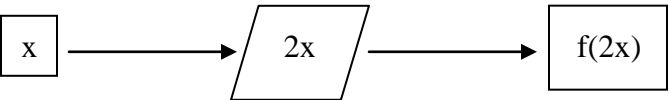
In	Out
x	$F(2x) = y$

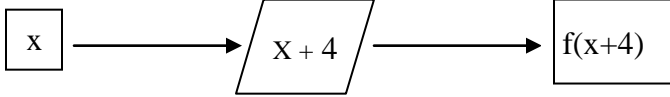
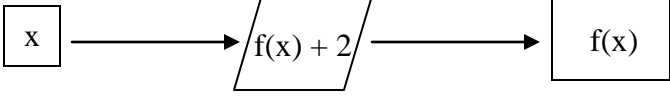


$$Y = f(x) + 2$$

x	f(x)	y = f(x) + 2

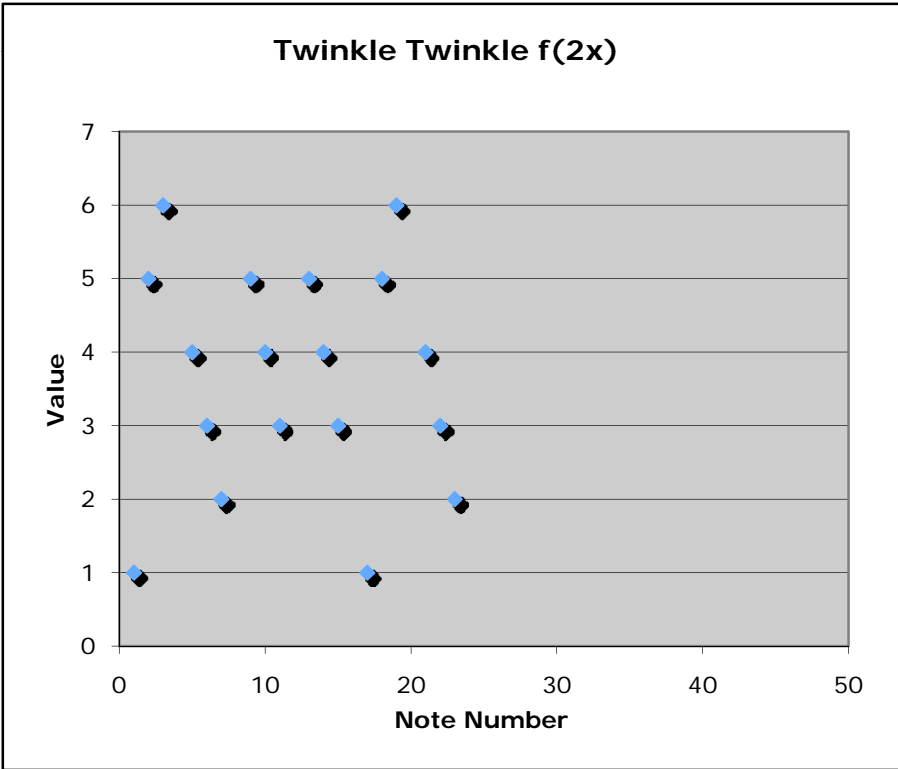
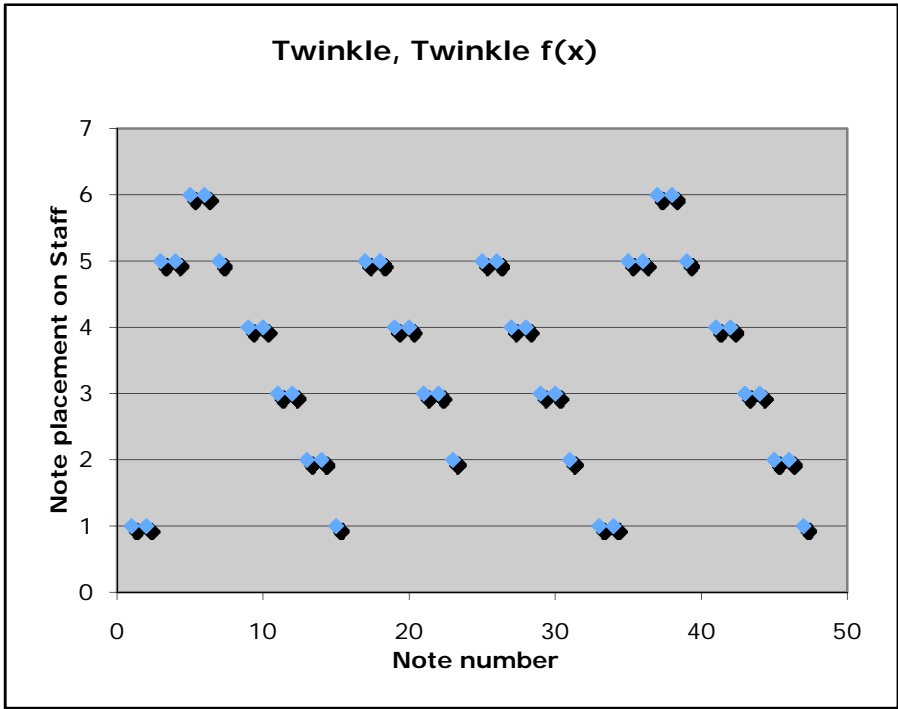


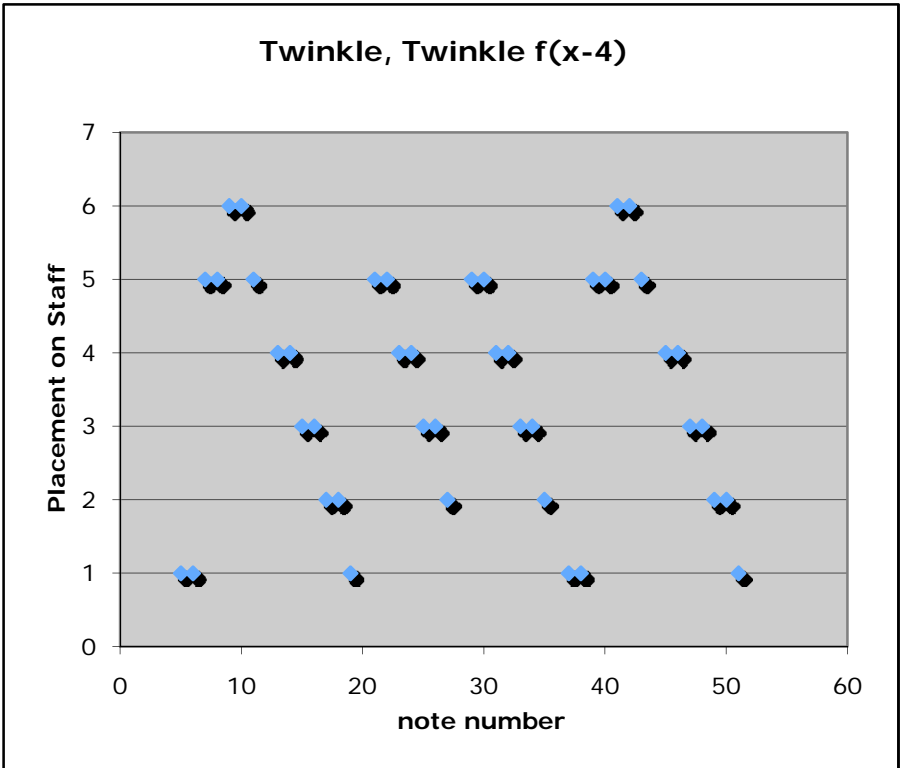
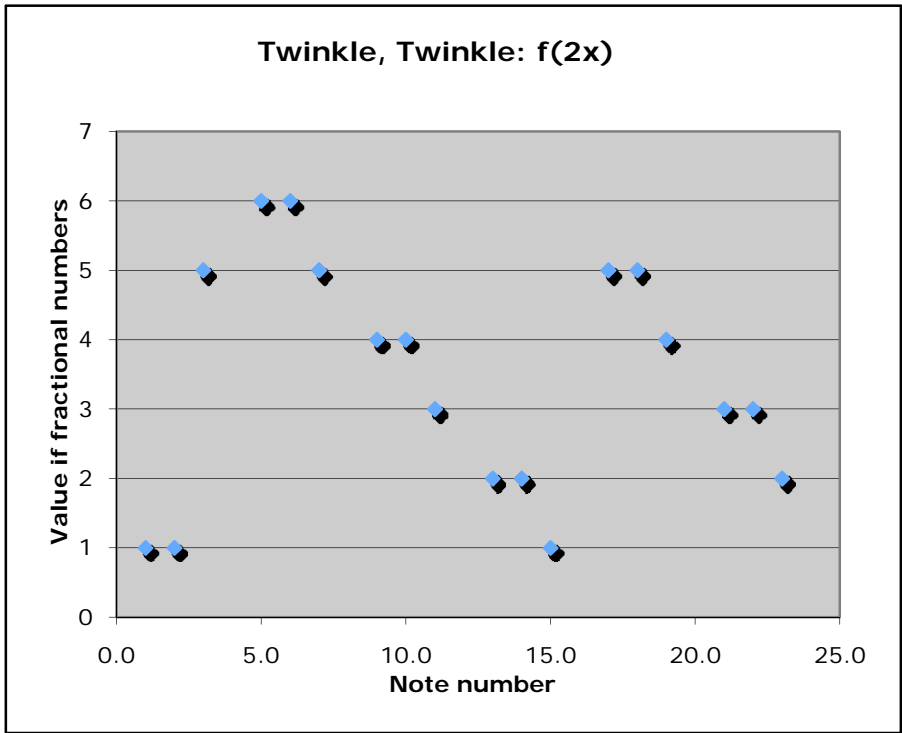
<p style="text-align: center;"><u>Notes</u></p> <p><b>Topic:</b> <b>Changing Domain of Musical Function (Translations)</b></p>	<p><b>Name:</b> _____</p> <p><b>Class:</b> _____</p> <p><b>Period:</b> _____</p> <p><b>Date:</b> _____</p>
<p style="text-align: center;"><b>Questions:</b></p> <p>How do you define a tune as a function?</p>	<p style="text-align: center;"><b>Notes:</b></p> <p>Given a function <math>f(x)</math> that is defined as musical notes of a tune. <math>f(1) = 1</math>, <math>f(2) = 1</math>, <math>f(3) = 5</math>, and so on. See the Twinkle Original input <math>x</math> values and output <math>f(x)</math> values.</p> 
<p>How can I slow down the tune?</p>	<p>If the input of the function <math>x</math> is multiplied by 0.5, then when <math>x = 1</math>, <math>f(0.5x) = f(0.5)</math>. So the domain will go from <math>(0.5)*1 \leq x \leq (0.5)*42</math>. The corresponding range will be experienced in twice the amount of time.</p> <p>Keep in mind that values of the output exist when the input is an integer value. Therefore, <math>f(0.5)</math> and <math>f(1.5)</math> do not have corresponding output values. <math>f(1) = 1</math>, <math>f(2) = 1</math>, <math>f(3) = 5</math>, and so on as in the first tune.</p> 
<p>How can I speed up the tune?</p>	<p>If the input of the function <math>x</math> is multiplied by 2, then when <math>x = 1</math>, <math>f(2x) = f(2) = 1</math>, when <math>x = 2</math>, <math>f(2x) = f(4) = 5</math>, and so on. As a result, the domain will go from <math>(2*1) \leq x \leq (2*42)</math>. The corresponding range will result as in the first tune, but it will be experienced in half the time.</p> <p>Keep in mind that values of the output exist when the input is an integer value. Since the input will be <math>2x</math>, the function will have values for only even input values. Therefore, the output for the odd input won't exist from the first tune.</p> 

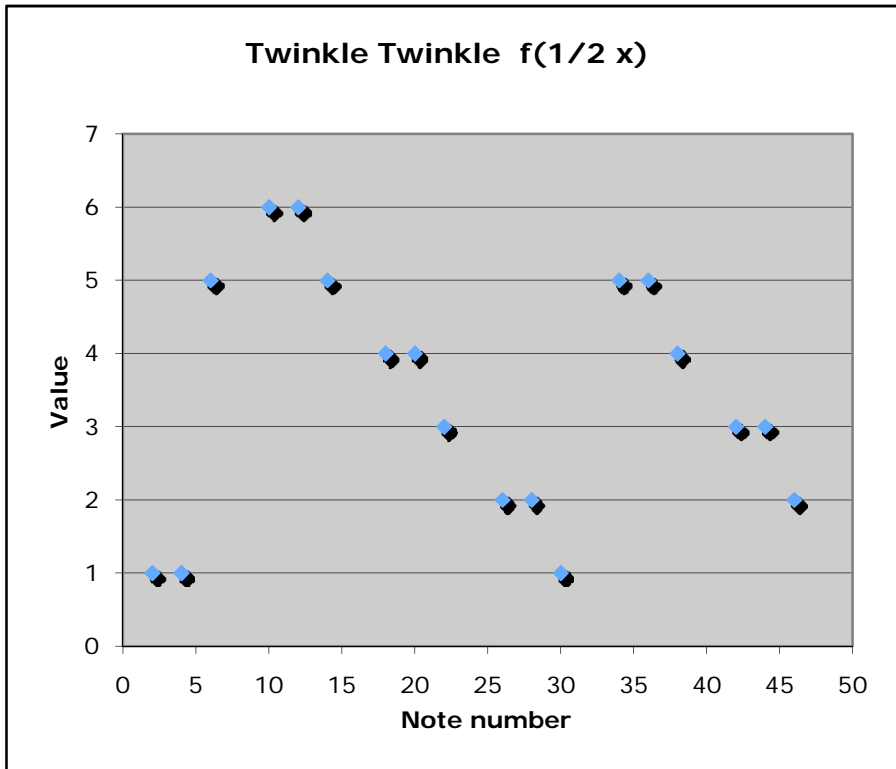
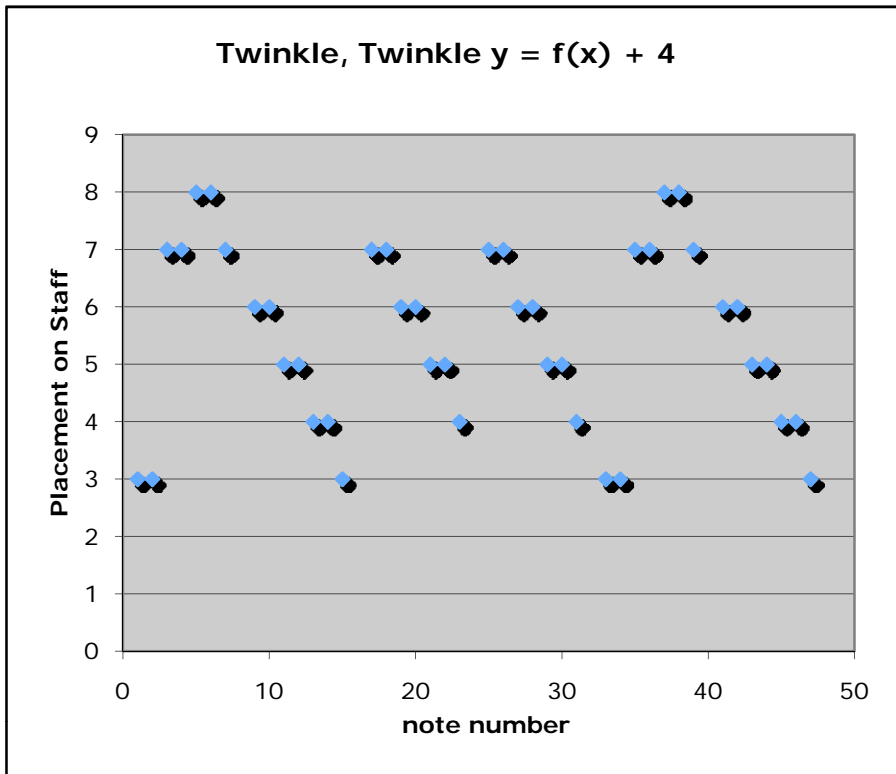
<p>How can I shift the tune so to play <i>a round</i>?</p>	<p>If the input of the function <math>x</math> has the constant value 4 added to it, then when <math>x = 1</math>, <math>f(x+4) = f(1+4) = f(5) = 6</math>. Therefore, the starting note of this musical experience will be the fifth note of the song.</p>  <pre> graph LR   A[x] --&gt; B[/x + 4/]   B --&gt; C[f(x+4)]   </pre>
<p>How can I change the key of the tune?</p>	<p>If the resulting output function has the constant value 2 added to it, then when <math>x = 1</math>, <math>f(1) = f(1) + 2 = 3</math>. When <math>x = 2</math>, <math>f(2) = f(2) + 2 = 3</math>. When <math>x = 3</math>, <math>f(3) = f(3) + 2 = 7</math>, and so on. Therefore, each note of the first tune will be shifted up two notes. ( This may not sound right to the ear since it does not take into consideration sharps or flats in the new key. )</p>  <pre> graph LR   A[x] --&gt; B[/f(x) + 2/]   B --&gt; C[f(x)]   </pre>
<p><b>Summary:</b></p>	<p><b>Treating songs as a function and applying changes to the input to analyze the output.</b></p>



note	value	note	note	value	Note	note	Value	Note	Value	note	value
1	1	0.5	1		2	1	1	5	1	1	3
2	1	1.0	2	1	4	2	5	6	1	2	3
3	5	1.5	3		6	3	6	7	5	3	7
4	5	2.0	4	1	8	4		8	5	4	7
5	6	2.5	5		10	5	4	9	6	5	8
6	6	3.0	6	5	12	6	3	10	6	6	8
7	5	3.5	7		14	7	2	11	5	7	7
9	4	4.5	9		18	9	5	13	4	9	6
10	4	5.0	10	6	20	10	4	14	4	10	6
11	3	5.5	11		22	11	3	15	3	11	5
12	3	6.0	12	6	24	12		16	3	12	5
13	2	6.5	13		26	13	5	17	2	13	4
14	2	7.0	14	5	28	14	4	18	2	14	4
15	1	7.5	15		30	15	3	19	1	15	3
17	5	8.5	17		34	17	1	21	5	17	7
18	5	9.0	18	4	36	18	5	22	5	18	7
19	4	9.5	19		38	19	6	23	4	19	6
20	4	10.0	20	4	40	20		24	4	20	6
21	3	10.5	21		42	21	4	25	3	21	5
22	3	11.0	22	3	44	22	3	26	3	22	5
23	2	11.5	23		46	23	2	27	2	23	4
25	5	12.5	25		50	25		29	5	25	7
26	5	13.0	26	2	52	26		30	5	26	7
27	4	13.5	27		54	27		31	4	27	6
28	4	14.0	28	2	56	28		32	4	28	6
29	3	14.5	29		58	29		33	3	29	5
30	3	15.0	30	1	60	30		34	3	30	5
31	2	15.5	31		62	31		35	2	31	4
33	1	16.5	33		66	33		37	1	33	3
34	1	17.0	34	5	68	34		38	1	34	3
35	5	17.5	35		70	35		39	5	35	7
36	5	18.0	36	5	72	36		40	5	36	7
37	6	18.5	37		74	37		41	6	37	8
38	6	19.0	38	4	76	38		42	6	38	8
39	5	19.5	39		78	39		43	5	39	7
41	4	20.5	41		82	41		45	4	41	6
42	4	21.0	42	3	84	42		46	4	42	6
43	3	21.5	43		86	43		47	3	43	5
44	3	22.0	44	3	88	44		48	3	44	5
45	2	22.5	45		90	45		49	2	45	4
46	2	23.0	46	2	92	46		50	2	46	4
47	1	23.5	47		94	47		51	1	47	3







## WORKSHEET 2: TWINKLE, TWINKLE TRANSFORMATIONS

Note x	f(x)
1	1
2	1
3	5
4	5
5	6
6	6
7	5
8	4
9	4
10	3
11	3
12	2
13	2
14	1
15	5
16	5
17	4
18	4
19	3
20	3
21	2
22	5
23	5
24	4
25	4
26	3
27	3
28	2
29	1
30	1
31	5
32	5
33	6
34	6
35	5
36	4
37	4
38	3
39	3
40	2
41	2
42	1

### Original Tune Number Assignment

When  $x = 1$ , the first tone assigned to the tune is given the value of 1. There are 42 tones assigned to this tune.

Domain:  $0 \leq x \leq 42$

Range:  $1 \leq f(x) \leq 6$

Graph the last two columns on a scatter plot on your calculator.

Note x	f(x)	0.5x	f(0.5x )
1	1	.5	
2	1	1	1
3	5	1.5	
4	5	2	1
5	6	2.5	
6	6	3	5
7	5	3.5	
8	4	4	5
9	4	4.5	
10	3	5	6
11	3	5.5	
12	2	6	6
13	2	6.5	
14	1	7	5
15	5	7.5	
16	5	8	4
17	4	8.5	
18	4	9	4
19	3	9.5	
20	3	10	3
21	2		
22	5		
23	5		
24	4		
25	4		
26	3		
27	3		
28	2		
29	1		
30	1		
31	5		
32	5		
33	6		
34	6		
35	5		
36	4		
37	4		
38	3		
39	3		
40	2		
41	2		
42	1	21	

Slow (Half as Fast) Version of the Original Tune

Domain:  $1 \leq x \leq 21$

Range:  $1 \leq f(0.5x) \leq 6$

Continue filling in the values for the 0.5x and the f(0.5x) columns.

Once the table is complete, apply the values of the new function to Finale Notepad and play your new arrangement of Twinkle Twinkle Little Star.

Graph the last two columns on a scatter plot on your calculator. Describe the difference you see graphically when compared with the original tune.

Describe what differences you hear when compared with the original tune.

Note x	f(x)	2x	f(2x)
1	1	2	1
2	1	4	5
3	5	6	6
4	5	8	4
5	6	10	3
6	6	12	2
7	5	14	1
8	4	16	5
9	4	18	4
10	3	20	3
11	3		
12	2		
13	2		
14	1		
15	5		
16	5		
17	4		
18	4		
19	3		
20	3		
21	2		
22	5		
23	5		
24	4		
25	4		
26	3		
27	3		
28	2		
29	1		
30	1		
31	5		
32	5		
33	6		
34	6		
35	5		
36	4		
37	4		
38	3		
39	3		
40	2		
41	2		
42	1	84	

Quick (Twice as Fast) Version of the Original Tune

Domain:  $1 \leq x \leq 84$

Range:  $1 \leq f(2x) \leq 6$

Continue filling in the values for the 2x and the f(2x) columns.

Once the table is complete, apply the values of the new function to Finale Notepad and play your new arrangement of Twinkle Twinkle Little Star.

Graph the last two columns on a scatter plot on your calculator. Describe the difference you see graphically when compared with the original tune.

Describe what differences you hear when compared with the original tune.

Note x	f(x)	x+4	f(x+4)
1	1	5	6
2	1	6	6
3	5	7	5
4	5	8	4
5	6	9	4
6	6	10	3
7	5	11	3
8	4	12	2
9	4	13	2
10	3	14	1
11	3	15	5
12	2	16	5
13	2	17	4
14	1	18	4
15	5	19	3
16	5	20	3
17	4		
18	4		
19	3		
20	3		
21	2		
22	5		
23	5		
24	4		
25	4		
26	3		
27	3		
28	2		
29	1		
30	1		
31	5		
32	5		
33	6		
34	6		
35	5		
36	4		
37	4		
38	3		
39	3		
40	2		
41	2		
42	1	46	

A Shifted (as in a Round) Version of the Original Tune

Domain:  $5 \leq x \leq 46$

Range:  $1 \leq f(2x) \leq 6$

Continue filling in the values for the x+4 and the f(x+4) columns.

Once the table is complete, apply the values of the new function to Finale Notepad and play your new arrangement of Twinkle Twinkle Little Star.

Graph the last two columns on a scatter plot on your calculator. Describe the difference you see graphically when compared with the original tune.

Describe what differences you hear when compared with the original tune.

Note x	f(x)	f(x)+2
1	1	3
2	1	3
3	5	7
4	5	7
5	6	8
6	6	8
7	5	7
8	4	6
9	4	6
10	3	5
11	3	
12	2	
13	2	
14	1	
15	5	
16	5	
17	4	
18	4	
19	3	
20	3	
21	2	
22	5	
23	5	
24	4	
25	4	
26	3	
27	3	
28	2	
29	1	
30	1	
31	5	
32	5	
33	6	
34	6	
35	5	
36	4	
37	4	
38	3	
39	3	
40	2	
41	2	
42	1	

A Shifted (as in a Key Change) Version of the Original Tune

Domain:  $1 \leq x \leq 42$

Range:  $3 \leq f(2x) \leq 8$

Continue filling in the values for the f(x)+2 columns.

Once the table is complete, apply the values of the new function to Finale Notepad and play your new arrangement of Twinkle Twinkle Little Star.

Graph the first and last columns on a scatter plot on your calculator. Describe the difference you see graphically when compared with the original tune.

Describe what differences you hear when compared with the original tune.

### WORKSHEET 3: MY PERSONAL SONG

Students might want to do this work on an excel worksheet.  
The notes for My Personal Song are:

X	F(x)

x	2x	f(2x)

x	f(x)	f(x) + 2

X	x - 4	f(x - 4)

x	2x	$\frac{1}{2} f(2x)$

## WORKSHEET 4: SINE CURVE TRANSFORMATIONS

Open the URL below and modify the four terms of a sine wave function. Sound waves generate sine functions and from them you can determine a variety of characteristic of those waves. From your explorations, you will fill out Cornell Notes on the effects of a, b, c, and d on the sine function. This will be one of the written assessments for this unit.

[http://www-personal.umich.edu/~yxl/486/ggb/sine\\_curve\\_transformations.html](http://www-personal.umich.edu/~yxl/486/ggb/sine_curve_transformations.html)





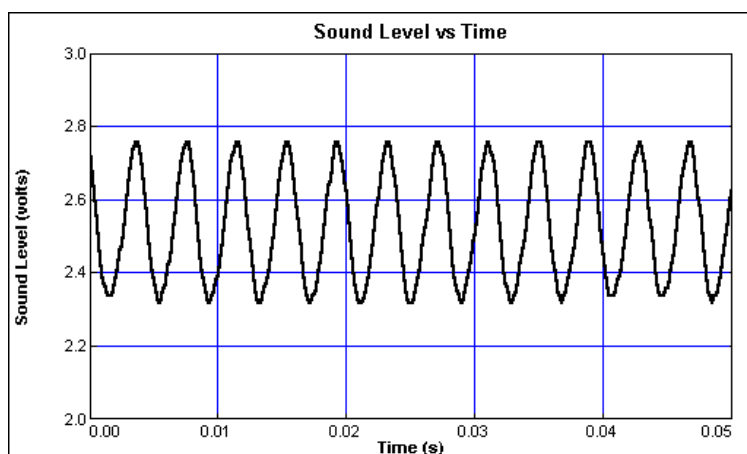
## WORKSHEET 4: TUNING FORKS

Sound waves consist of a series of air pressure variations. A Microphone diaphragm records these variations by moving in response to the pressure changes. The diaphragm motion is then converted to an electrical signal. Using a Microphone and a computer interface, you can explore the properties of common sounds.

The first property you will measure is the *period*, or the time for one complete cycle of repetition. Since period is a time measurement, it is usually written as  $T$ . The reciprocal of the period ( $1/T$ ) is called the *frequency*,  $f$ , the number of complete cycles per second. Frequency is measured in hertz (Hz).  $1 \text{ Hz} = 1 \text{ s}^{-1}$ .

A second property of sound is the *amplitude*. As the pressure varies, it goes above and below the average pressure in the room. The maximum variation above or below the pressure mid-point is called the amplitude. The amplitude of a sound is closely related to its loudness.

When two sound waves overlap, their air pressure variations will combine. For sound waves, this combination is additive. We say that sound follows the principle of *linear superposition*. Beats are an example of superposition. Two sounds of nearly the same frequency will create a distinctive variation of sound amplitude, which we call beats. You can study this phenomenon with a Microphone, lab interface, and computer.



## OBJECTIVES

- Measure the frequency and period of sound waves from tuning forks.
- Measure the amplitude of sound waves from tuning forks.
- Observe beats between the sound of two tuning forks.

## MATERIALS

Windows PC

Logger *Pro*

LabPro


2 tuning forks or electronic keyboard


Vernier Microphone

## PRELIMINARY QUESTIONS

1. Why are instruments tuned before being played as a group? In which ways do musicians tune their instruments?
2. Given that sound waves consist of series of air pressure increases and decreases, what would happen if an air pressure increase from one sound wave was located at the same place and time as a pressure decrease from another of the same amplitude?

## PROCEDURE

1. Connect the Vernier Microphone to Channel 1 of the LabPro.
2. Open the file in the Experiment 21 folder of *Physics with Computers*. The computer will take data for just 0.05 s to display the rapid pressure variations of sound waves. The vertical axis corresponds to the variation in air pressure and the units are arbitrary.
3. Produce a sound with a tuning fork or keyboard, hold it close to the Microphone and click . The data should be sinusoidal in form, similar to the sample on the front page of this lab. If you are using a tuning fork, strike it against a soft object such as a rubber mallet or the rubber sole of a shoe. Striking it against a hard object can damage it. If you strike the fork too hard or too softly, the waveform may be too rough; try again.

4. Note the appearance of the graph. Count and record the number of complete cycles shown after the first peak in your data.
5. Click the Examine button, . Drag the mouse across the graph and record the times for the first and last peaks of the waveform. Divide the difference,  $\Delta t$ , by the number of cycles to determine the period of the tuning fork.
6. Calculate the frequency of the tuning fork in Hz and record it in your data table.
7. Drag the mouse across the graph and record the maximum and minimum y values for an adjacent peak and trough.
8. Calculate the amplitude of the wave by taking half of the difference between the maximum and minimum y values. Record the values in your data table.
9. Make a sketch of your graph or print the graph.
10. Save your data by choosing Store Latest Run from the Data menu. Hide the run by choosing Hide Run ► Run 1 from the Data menu.
11. Repeat Steps 3 – 9 for the second frequency. Store the latest run. It will be stored as Run 2. Then hide Run 2.

## DATA TABLE

### Part I: Simple Waveforms

Tuning fork or note	Number of cycles	First maximum (s)	Last maximum (s)	$\Delta t$ (s)	Period (s)	Calculated frequency (Hz)

Tuning fork or note	Peak (V)	Trough (V)	Amplitude (V)

Tuning fork or note	Parameter A (V)	Parameter B (s <sup>-1</sup> )	$f = B/2\pi$ (Hz)

## ANALYSIS



### Part I: Simple Waveforms


1. In the following analysis, you will see how well a sine function model fits the data. The displacement of the particles in the medium carrying a periodic wave can be modeled with a sinusoidal function. Your textbook may have an expression resembling this one:

$$y = A \sin(2\pi f t)$$

In the case of sound, a longitudinal wave, the  $y$  refers to the change in air pressure that makes up the wave.  $A$  is the amplitude of the wave (a measure of loudness), and  $f$  is the frequency. Time is represented with  $t$  and the sine function requires a factor of  $2\pi$  when evaluated in radians.

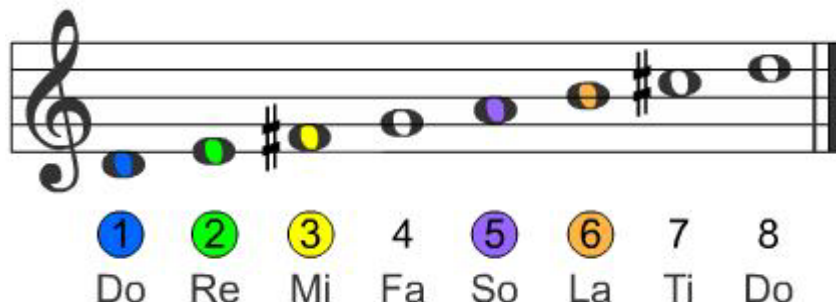
Logger *Pro* will fit the function  $y = A * \sin(B*x + C) + D$  to experimental data.  $A$ ,  $B$ ,  $C$ , and  $D$  are parameters (numbers) that Logger *Pro* reports after a fit. This function is more complicated than the textbook model, but the basic sinusoidal form is the same. Comparing terms, listing the textbook model's terms first, the amplitude  $A$  corresponds to the fit term  $A$ , and  $2\pi f$  corresponds to the parameter  $B$ . The time  $t$  is represented by the  $x$ , Logger *Pro*'s  $x$ -axis. The new parameters  $C$  and  $D$  shift the fitted function left-right and up-down, respectively and are necessary to obtain a good fit. Only the parameters  $A$  and  $B$  are important to this experiment. In particular, the numeric value of  $B$  allows you to find the

frequency  $f$  using  $B = 2\pi f$ . Choose Show ▶ Run 1 from the Data menu to show the waveform from the first tone. Hide the other runs. Click the Curve Fit button, , select “ $A*\sin(B*x + C) + D$ ” from the list of models and “Sound Level Run 1” from the Perform Fit On list. Click  to perform the curve fit.

2. Click  to return to the graph. The model and its parameters appear in a floating box in the upper left corner of the graph. Record the parameters  $A$  and  $B$  of the model in your data table.
3. Since  $B$  corresponds to  $2\pi f$  in the curve fit, use the curve fit information to determine the frequency. Enter the value in your data table. Compare this frequency to the frequency calculated earlier. Which would you expect to be more accurate? Why?
4. Compare the parameter  $A$  to the amplitude of the waveform.
5. Hide Run 1 and show Run 2, the waveform of the second tone. Repeat Steps 1 – 4 for Run 2.

## TUBULAR GLOCKENSPIEL

### Building a 5-note Glockenspiel



You will build a tubular glockenspiel using metal tubing and determining the notes from each team.

You will receive one of five tubes. Using the information gained from the tuning fork activity determine the note for your tube. Arrange the tubes and play the song Mary had a little lamb.

Notes are F# E D E F# F# E E E F# A A

Frequencies for the notes are on the attached document.

#### ***Teacher preparation***

Get ten feet of 1/2-inch Electrical Metallic Tubing (EMT) pipe — also referred to as Electrical Conduit, or EMT Conduit.

1) With a metal pipe cutter (not a hacksaw), cut the tubing into the five sections listed on the left (some full-service hardware stores will do this for you).

2) Arrange your tubes as shown so that the longest is on the bottom and the shortest is on top.

3) String the tubes together (simple knots will do) along both sides so there are 2 inches between each tube. You may also place the tubes on two pieces of felt or foam rubber.

4) Try different "mallets" (metal/wood/plastic) to obtain a variety of timbres.

	<u>Inches</u>	<u>Centimeters</u>	
D	11 15/16	30.4	1
E	11 1/4	28.6	2
F#	10 9/16	26.8	3
A	9 11/16	24.6	5
B	9 1/8	23.2	6

Adapted from Phil Tulga site <http://www.philtulga.com/index.html>  
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# Physics of Music - Notes

## Frequencies for equal-tempered scale

This table created using  $A_4 = 440$  Hz  
 Speed of sound = 345 m/s = 1130 ft/s = 770 miles/hr

("Middle C" is  $C_4$ )

Note	Frequency (Hz)	Wavelength (cm)
$C_0$	16.35	2100.
$C^\#_0/D^b_0$	17.32	1990.
$D_0$	18.35	1870.
$D^\#_0/E^b_0$	19.45	1770.
$E_0$	20.60	1670.
$F_0$	21.83	1580.
$F^\#_0/G^b_0$	23.12	1490.
$G_0$	24.50	1400.
$G^\#_0/A^b_0$	25.96	1320.
$A_0$	27.50	1250.
$A^\#_0/B^b_0$	29.14	1180.
$B_0$	30.87	1110.
$C_1$	32.70	1050.
$C^\#_1/D^b_1$	34.65	996.
$D_1$	36.71	940.
$D^\#_1/E^b_1$	38.89	887.
$E_1$	41.20	837.
$F_1$	43.65	790.
$F^\#_1/G^b_1$	46.25	746.
$G_1$	49.00	704.
$G^\#_1/A^b_1$	51.91	665.
$A_1$	55.00	627.
$A^\#_1/B^b_1$	58.27	592.
$B_1$	61.74	559.

$C_2$	65.41	527.
$C^{\#}_2/D^b_2$	69.30	498.
$D_2$	73.42	470.
$D^{\#}_2/E^b_2$	77.78	444.
$E_2$	82.41	419.
$F_2$	87.31	395.
$F^{\#}_2/G^b_2$	92.50	373.
$G_2$	98.00	352.
$G^{\#}_2/A^b_2$	103.83	332.
$A_2$	110.00	314.
$A^{\#}_2/B^b_2$	116.54	296.
$B_2$	123.47	279.
$C_3$	130.81	264.
$C^{\#}_3/D^b_3$	138.59	249.
$D_3$	146.83	235.
$D^{\#}_3/E^b_3$	155.56	222.
$E_3$	164.81	209.
$F_3$	174.61	198.
$F^{\#}_3/G^b_3$	185.00	186.
$G_3$	196.00	176.
$G^{\#}_3/A^b_3$	207.65	166.
$A_3$	220.00	157.
$A^{\#}_3/B^b_3$	233.08	148.
$B_3$	246.94	140.
$C_4$	261.63	132.
$C^{\#}_4/D^b_4$	277.18	124.
$D_4$	293.66	117.
$D^{\#}_4/E^b_4$	311.13	111.
$E_4$	329.63	105.
$F_4$	349.23	98.8
$F^{\#}_4/G^b_4$	369.99	93.2
$G_4$	392.00	88.0
$G^{\#}_4/A^b_4$	415.30	83.1
$A_4$	440.00	78.4
$A^{\#}_4/B^b_4$	466.16	74.0

B <sub>4</sub>	493.88	69.9
C <sub>5</sub>	523.25	65.9
C <sup>#</sup> <sub>5</sub> /D <sup>b</sup> <sub>5</sub>	554.37	62.2
D <sub>5</sub>	587.33	58.7
D <sup>#</sup> <sub>5</sub> /E <sup>b</sup> <sub>5</sub>	622.25	55.4
E <sub>5</sub>	659.26	52.3
F <sub>5</sub>	698.46	49.4
F <sup>#</sup> <sub>5</sub> /G <sup>b</sup> <sub>5</sub>	739.99	46.6
G <sub>5</sub>	783.99	44.0
G <sup>#</sup> <sub>5</sub> /A <sup>b</sup> <sub>5</sub>	830.61	41.5
A <sub>5</sub>	880.00	39.2
A <sup>#</sup> <sub>5</sub> /B <sup>b</sup> <sub>5</sub>	932.33	37.0
B <sub>5</sub>	987.77	34.9
C <sub>6</sub>	1046.50	33.0
C <sup>#</sup> <sub>6</sub> /D <sup>b</sup> <sub>6</sub>	1108.73	31.1
D <sub>6</sub>	1174.66	29.4
D <sup>#</sup> <sub>6</sub> /E <sup>b</sup> <sub>6</sub>	1244.51	27.7
E <sub>6</sub>	1318.51	26.2
F <sub>6</sub>	1396.91	24.7
F <sup>#</sup> <sub>6</sub> /G <sup>b</sup> <sub>6</sub>	1479.98	23.3
G <sub>6</sub>	1567.98	22.0
G <sup>#</sup> <sub>6</sub> /A <sup>b</sup> <sub>6</sub>	1661.22	20.8
A <sub>6</sub>	1760.00	19.6
A <sup>#</sup> <sub>6</sub> /B <sup>b</sup> <sub>6</sub>	1864.66	18.5
B <sub>6</sub>	1975.53	17.5
C <sub>7</sub>	2093.00	16.5
C <sup>#</sup> <sub>7</sub> /D <sup>b</sup> <sub>7</sub>	2217.46	15.6
D <sub>7</sub>	2349.32	14.7
D <sup>#</sup> <sub>7</sub> /E <sup>b</sup> <sub>7</sub>	2489.02	13.9
E <sub>7</sub>	2637.02	13.1
F <sub>7</sub>	2793.83	12.3
F <sup>#</sup> <sub>7</sub> /G <sup>b</sup> <sub>7</sub>	2959.96	11.7
G <sub>7</sub>	3135.96	11.0
G <sup>#</sup> <sub>7</sub> /A <sup>b</sup> <sub>7</sub>	3322.44	10.4
A <sub>7</sub>	3520.00	9.8

$A^{\#}_7/B^b_7$	3729.31	9.3
$B_7$	3951.07	8.7
$C_8$	4186.01	8.2
$C^{\#}_8/D^b_8$	4434.92	7.8
$D_8$	4698.64	7.3
$D^{\#}_8/E^b_8$	4978.03	6.9

(To convert lengths in cm to inches, divide by 2.54)

## MAKING MUSIC WITH WINE GLASSES

Make some interesting and eerie music with wineglasses.

### What is going on?

These effects are all to do with a principle called resonance. Some things have a speed that they vibrate at really well, for example a swing - just by wobbling your legs at the right speed you can build up a great big swing and have lots of fun.



Your finger is doing something similar to the glass, as you slide it around, it will tend to stick, then slip, then stick then slip. Some glasses have a speed at which they will vibrate really really well, if this sticking and slipping is at about the same speed as this the vibration will build up enough that you can start to hear it as the eerie note.

Some glasses are better at vibrating than others, cheap ones tend to have minute flaws in their structure that rub against one another while the glass is vibrating causing it to lose energy - a bit like putting your feet down on a swing. In this case you are never going to get it to vibrate well. The best glasses are large crystal wineglasses that haven't been decorated.

If you add water to the glass it essentially makes the glass heavier so it takes it longer to vibrate back and forth so vibration is slower and therefore the pitch is lower.

It may make you go mad

Instruments based on this very principle were very popular at the beginning of the 19th century, called Glass Harmonicas or Armonicas they consisted of a series of tuned glass bowls which the player could rub to produce different notes. Although after a while it was believed that the beautiful eerie sounds could make people particularly women go mad...

Although it was later discovered that they were marking the notes with lead paint, and the players were licking their fingers to play the instrument, and therefore eating lots of poisonous lead paint, which was probably more of the problem.

### **What you need**

- A few wineglasses - ideally some cheap and some expensive
- Some Water
- A finger

### **What to do**

- Try pinging the glasses, see how long they keeps ringing for
- Pick the one that rang for the longest.
- Wet your finger, and then gently rub it around the rim of the glass - don't press too hard or you could break the glass.

### **What may happen?**

You should find that any glass that will ring for more than 3-4 seconds will start to vibrate and produce an eerie note as you move your finger around it. It may take a while to get the knack but it should work.

Different glasses will produce different notes and will work differently well, you should find that the glasses that ring for longest after you ping them are easiest to play.

If you add water, you should find that the pitch of the note goes down.

Using the same method from the tuning fork activity and tubular glockenspiel activity tune the glasses to the following notes.

D F# A E B

When finished play the following song:

D F# A A B A F# D E F# F# E D

*Modified from a web article Written by [Dave Ansell](#)*