

## A Framework for Procedural Understanding: Theory into Practice

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### Discussion prompts

(these will arise from today's presentation):

#### Part 1: Theory and Motivation for TfU

1. To what extent can we infer understanding from skillful execution? and How well do you think *your* students 'understand' mathematics?

2. Brainstorm a list of procedures from algebra (or the subject you teach). Then brainstorm a list of concepts.

Procedures:

E.g. Solve  $ax+b = c$

Concepts:

E.g. Slope is the rate of change

3. Select one of the procedures you listed above. What sorts of activities would facilitate growth along either (or both) the aptitude dimension or the depth dimension?

4. Find the maximum of  $f(x) = -2x^2 + 4x + 3$ .

a. What sort of answer should you expect?

b. How else could you find the maximum?

c. Why does your method work?

d. Is your answer reasonable? How could you check?

e. Of the available methods, (how do you decide) which one is the 'best'?

f. What types of problems can I solve with this procedure?

5. What did you learn from Part 1 that will stick with you?

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### Part 2: TfU Implementations

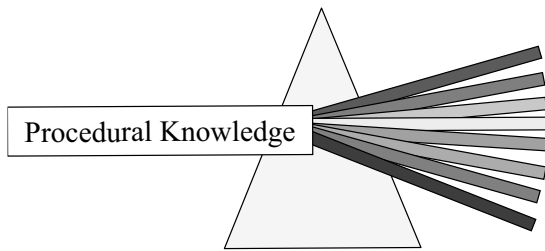
6. What implications do the two studies described in Part 2 have for your institution?

### Part 3: (Re)focusing the curriculum:

7. 'Generative topics' are central to one or more disciplines, interesting and accessible, and provide opportunities for multiple connections. These should form the core of the curriculum. What do you see as 'generative topics' from the classes you teach?
8. Describe one or two 'throughlines' (broad understanding goals) that wind their way through your courses?
9. If your goals include teaching for understanding, your assessment system must align with that goal. What are some ways you might incorporate 'performances of understanding' into your own assessment system?
10. Other Notes from Part 3?

Closing Notes: What are your take-home points from today's presentation?

# Framework for Procedural Understanding (or, Eight Powerful Questions)



- 1a) What is the goal of the procedure?
- 1b) What sort of answer should I expect?
- 2a) How do I carry out the procedure?
- 2b) What other procedures could I use?
- 3) Why does the procedure work?
- 4) How can I verify my answer?
- 5) When is this the "best" procedure to use?
- 6) What else can I use this procedure to do?

Adapted from NCTM's Navigating through Algebra in Grades 9-12

View the following problems through the lens of the Framework:

Solve:  $2x - 1 = 6$

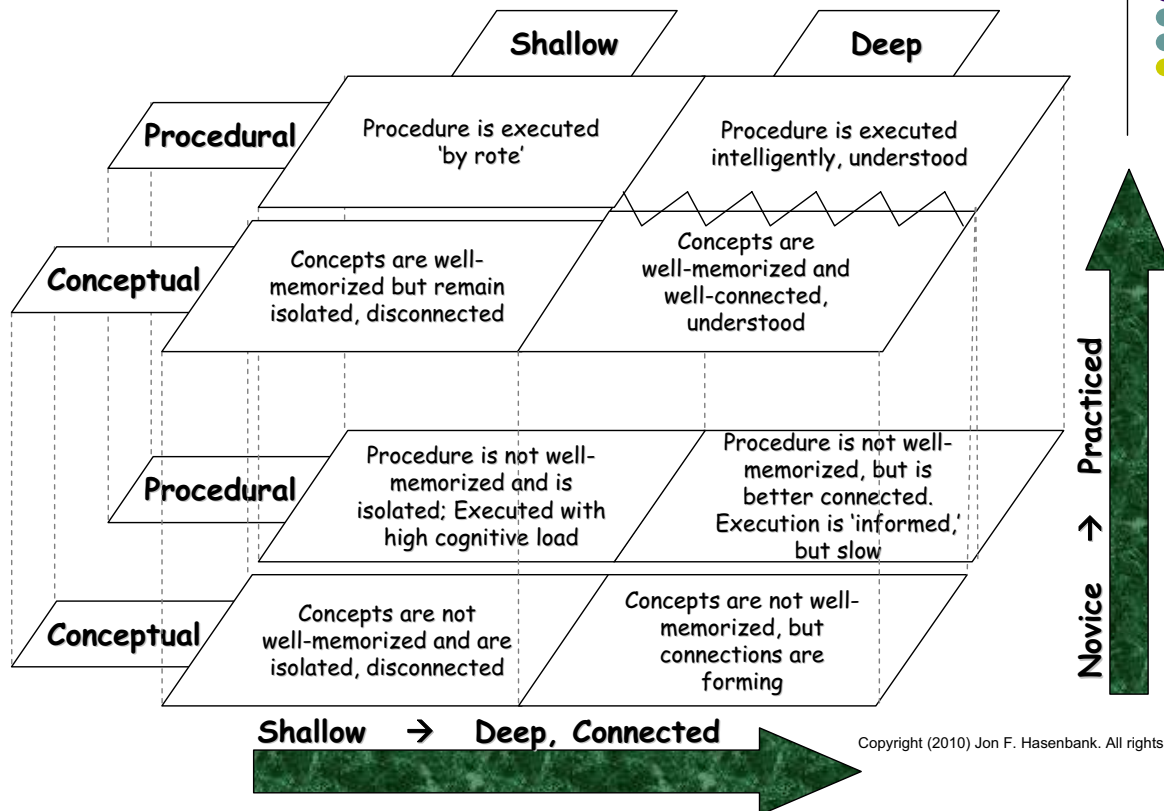
Solve:  $|x - 3| < 5$

Solve:  $x^2 + 3x - 4 = 0$

Solve:  $2x + 3y < 6$

Solve:  $3x + 4y = 14$  and  $y = -(5/2)x$

## Three Dimensions of Knowledge



0113

1. What is the value of the expression below when  $x = -4$ ?  
 $(x+1)(x+2)$

$(-4+1)(-4+2)$   
 $(-3)(-2)$   
 $6$

	-4	
		2

2. If we could change  $x$  to equal something else, could  $(x+1)(x+2)$  ever be a negative number? Explain.

Yes. If  $x$  is negative, the answer will be too.

3. Solve this equation for  $x$ . Show your steps.

$$5x - 6 = 4x + 3$$

$$\frac{5x}{5x} - 9 = \frac{4x}{5x} + \frac{3}{5x}$$

$$-9 = \frac{8x}{5x} + 11.25 = x$$

4. Describe two ways you could check your answer to the previous question.

One way to check my answer: Plug in a number to  $x$   
 Another way to check my answer: create a table or graph.

15. If  $3x + 4y = 14$  and  $y = -\frac{5}{2}x$ , what is the value of  $x$ ?

$x = \text{between}$   
 $-5 \leq -6$

16. What does the answer to the previous problem tell you about the graphs of the two equations?

They are parallel

0126

1. What is the value of the expression below when  $x = -4$ ?  
 $(x+1)(x+2)$

$(-4+1)(-4+2)$   
 $(-3)(-2)$   
 $6$

2. If we could change  $x$  to equal something else, could  $(x+1)(x+2)$  ever be a negative number? Explain.

$(-10+1)(10+2)$   
 $-9 \times 8$  No

3. Solve this equation for  $x$ . Show your steps.

$$5x\sqrt{6} + 4x + 3$$

$$-4x\sqrt{6} + 4x + 6$$

$$\frac{5x\sqrt{6}}{5x\sqrt{6}} + \frac{4x}{5x\sqrt{6}} + \frac{3}{5x\sqrt{6}}$$

$$\sqrt{6}x = 9$$

$$x = -1$$

4. Describe two ways you could check your answer to the previous question.

One way to check my answer: put  $-1$  in for  $x$   
 Another way to check my answer: redo your work

15. If  $3x + 4y = 14$  and  $y = -\frac{5}{2}x$ , what is the value of  $x$ ?

$3x + 4(-\frac{5}{2}x) = 14$   
 $3x - 10x = 14$   
 $-7x = 14$   
 $x = -2$

16. What does the answer to the previous problem tell you about the graphs of the two equations?

they will meet

0335

1. What is the value of the expression below when  $x = -4$ ?

$$\frac{(x+1)(x+2)}{(-4+1)(-4+2)}$$

6

2. If we could change  $x$  to equal something else, could  $(x+1)(x+2)$  ever be a negative number? Explain.

No, because the result would either be two negatives, two positives, or a number times 0, so it couldn't be negative.

3. Solve this equation for  $x$ . Show your steps.

$$\begin{array}{r} 5x - 6 = 4x + 3 \\ -4x \quad -6 \\ \hline -x = 9 \end{array}$$

4. Describe two ways you could check your answer to the previous question.

One way to check my answer:

to plug 9 into  $x$  as the equation and see if it works

Another way to check my answer:

to kind of guess-check and see if it would look like it would make sense

15. If  $3x + 4y = 14$  and  $y = -\frac{5}{2}x$ , what is the value of  $x$ ?

$$x = -8$$

16. What does the answer to the previous problem tell you about the graphs of the two equations?

The graphs intersect at a point that would be negative and  $x > y$ .

0435

1. What is the value of the expression below when  $x = -4$ ?

$$\frac{(x+1)(x+2)}{(-4+1)(-4+2)}$$

6

2. If we could change  $x$  to equal something else, could  $(x+1)(x+2)$  ever be a negative number? Explain.

Yes, only if  $x$  was changed again to a negative

3. Solve this equation for  $x$ . Show your steps.

$$\begin{array}{r} 5x - 6 = 4x + 3 \\ -4x \quad -6 \\ \hline x - 6 = 3 \\ +6 \quad +6 \\ \hline x = 9 \end{array}$$

4. Describe two ways you could check your answer to the previous question.

One way to check my answer:

plug  $x=9$  into the  $x$  places in the problem

Another way to check my answer:

Do the problem a different way to see if you get the same answer

15. If  $3x + 4y = 14$  and  $y = -\frac{5}{2}x$ , what is the value of  $x$ ?

$$3x + 4(-2.5) = 14$$

$$3x + 10 = 14$$

$$\frac{3x}{3} = \frac{4}{3}$$

$$x = \frac{4}{3} = 1\frac{1}{3}$$

16. What does the answer to the previous problem tell you about the graphs of the two equations?

That they are on graphs

0630

1. What is the value of the expression below when  $x = -4$ ?  
 $(x+1)(x+2)$

$(-4+1)(-4+2)$   
 $10-8-4+2$   
 $6$

2. If we could change  $x$  to equal something else, could  $(x+1)(x+2)$  ever be a negative number? Explain.

NO even if you put in a very high, or very low number for  $x$ , it would be a positive answer.

3. Solve this equation for  $x$ . Show your steps.

$5x - 6 = 4x + 3$   
 $x = 9$

4. Describe two ways you could check your answer to the previous question.

One way to check my answer: you can put what you think  $x$  is in the original problem & see if it's right.  
Another way to check my answer: you could go back & recheck your steps.

15. If  $3x + 4y = 14$  and  $y = -\frac{5}{2}x$ , what is the value of  $x$ ?

$3x + 4(-\frac{5}{2}x) = 14$   
 $3x - 10x = 14$   
 $-7x = 14$   
 $x = -2$

16. What does the answer to the previous problem tell you about the graphs of the two equations?

This shows that the line will be negative.



0606

1. What is the value of the expression below when  $x = -4$ ?  
 $(x+1)(x+2)$

$(-4+1)(-4+2)$   
 $-16-12+2$

2. If we could change  $x$  to equal something else, could  $(x+1)(x+2)$  ever be a negative number? Explain.

yes because there would only be 1 negative and that would make the answer - (you would put a - in for  $x$ )

3. Solve this equation for  $x$ . Show your steps.

$5x - 6 = 4x + 3$   
 $5x - 6 = 4x + 3$   
 $-4x + 6 = -4x + 6$   
 $x = 9$

4. Describe two ways you could check your answer to the previous question.

One way to check my answer: plug  $x$  in to the equation  
Another way to check my answer: you could solve it and put 9 in for  $x$

15. If  $3x + 4y = 14$  and  $y = -\frac{5}{2}x$ , what is the value of  $x$ ?

$3x + 4(-\frac{5}{2}x) = 14$   
 $6x + 8(-8) = 28$   
 $6x - 40x = 28$   
 $-34x = 28$   
 $x = \frac{28}{-34}$   
 $x = \frac{14}{-17}$

16. What does the answer to the previous problem tell you about the graphs of the two equations?

one is a negative graph and one is a positive one

1547

1. What is the value of the expression below when  $x = -4$ ?

$$\frac{(x+1)(x+2)}{(-4+1)(-4+2)}$$

$$\frac{(-3)(-2)}{(-3)(-2)}$$

$$\boxed{6}$$

2. If we could change  $x$  to equal something else, could  $(x+1)(x+2)$  ever be a negative number? Explain.

Yes. If  $x = -1.7$ , it would be negative.

0789

1. What is the value of the expression below when  $x = -4$ ?

$$\frac{(x+1)(x+2)}{-3 \cdot 2}$$

$$\boxed{6}$$

2. If we could change  $x$  to equal something else, could  $(x+1)(x+2)$  ever be a negative number? Explain. No, because you would have to use -1 and that would change  $(x+1)=0$  and anything times zero equals zero.

3. Solve this equation for  $x$ . Show your steps.

$$\frac{5x-6=4x+3}{-4x+6 = -4x+6}$$

$$5x$$

$$x=9$$

4. Describe two ways you could check your answer to the previous question.

One way to check my answer: I checked  $9$  into the  $x$  position.

Another way to check my answer: I should redo the problem.

15. If  $3x+4y=14$  and  $y=-\frac{5}{2}x$ , what is the value of  $x$ ?

$$x = -2$$

16. What does the answer to the previous problem tell you about the graphs of the two equations?

they are parallel they will not intersect.

3. Solve this equation for  $x$ . Show your steps.

$$\frac{5x-6=4x+3}{+6 \quad +6}$$

$$\frac{5x=4x+9}{-4x \quad -4x}$$

$$\boxed{x=9}$$

4. Describe two ways you could check your answer to the previous question.

One way to check my answer:

put  $9$  in for  $x$

Another way to check my answer:

work backwards.

15. If  $3x+4y=14$  and  $y=-\frac{5}{2}x$ , what is the value of  $x$ ?

$$3x+4\left(-\frac{5}{2}x\right)=14$$

$$3x-10=14$$

$$3x=24$$

$$\frac{3x}{3}=\frac{24}{3}$$

$$\boxed{x=8}$$

16. What does the answer to the previous problem tell you about the graphs of the two equations?

They have the same  $x$ .

OVERARCHING UNDERSTANDING GOALS, OR THROUGH-LINES  
*"The things I want my students to understand after this course or year are..."*  
Statements such as "Students will understand..."

GENERATIVE TOPIC  
*"I consider this to be a generative topic because..."*

UNIT-LONG UNDERSTANDING GOALS  
*"Students will understand..."* and  
*"The questions I'd like my students to be able to answer are..."*

PERFORMANCES OF UNDERSTANDING  
*"Students will build toward achieving the understanding goals by..."*

ONGOING ASSESSMENT  
*"Students will get feedback on their performances by..."*  
(That is, how will they know how well they are doing?)  
*"The criteria for each performance will be..."*