Module 4:
Linear Programming

This material corresponds to chapter 4 of the textbook, *For All Practical Purposes*

LINEAR PROGRAMMING
CHAPTER 4

TIME FRAME: 12 days

ENDURING UNDERSTANDINGS:

When determining an optimal solution, considering all constraints will help identify possible profitable solutions. Helping businesses allocate the resources it has on hand in order to make a profit is the main focus of management science in the business world.

ESSENTIAL (ASSESSMENT) QUESTIONS:

1. What is the relationship between constraint equation and the optimal production equation?
2. What does it mean to a business to have a solution to a Liner Programming problem?
3. Why would businesses use the NW corner rule to solve transformation problems?

CRMS:

Reasoning/Problem-Solving: 1.1, 1.2, 1.3
Communication: 2.1, 2.2, 2.3
Connections: 3.1, 3.2, 3.3, 3.4
Algebra: 7.3c, d, f, j
Functions: 8.1b, 8.2a, b, 8.3c, 8.4a-d

AT THE END OF THE MODULE STUDENTS WILL KNOW AND BE ABLE TO:

1. Identify variables, constraint and profit equations.
2. Draw the feasible region.
3. Analyze corner points to determine production policy.
4. Solve transportation problems using the NW corner rule.

PREREQUISITE KNOWLEDGE/SKILLS:

Graphing linear equations and graphing linear inequalities, finding points of intersection and solving literal equations.

PRE-ASSESSMENT ACTIVITY:

Linear Programming Pre-Assessment Activity

ACTIVITIES:

Worksheet 1: **Systems of Equations and Inequalities Practice**
Worksheet 2: **The Corner Point Principle**
Numb3rs Activity: **Branch and Bound**
Worksheet 3: **Mixture problems**
Worksheet 4: **Northwest Corner and Transportation Problems**

POST-ASSESSMENT:

Give a written test using problems from the test bank. Be careful not to make the assessment too long. It may help to have students do intermediate steps for a few problems then have one problem that is solved completely. A sample exam is included that may be used as a template or as a study guide for students, at the instructor's discretion.

RESOURCES:

FAPP Instructor Test Bank CD-ROM
Project TIME DVD (*Numb3rs* clip from "The Mole")
TI-84 Graphing Calculators

DAILY PLAN

Remember to pick at least two days each week to begin class with a warm-up question from the "Algebra Skills Review" found in the Algebra Skills module to this Teacher Resource Manual.

DAYS 1 - 2

- Introductory problem: Have students discuss possible solutions in groups.
- Discussion: Share out ideas from groups.
- **Linear Programming Pre-Assessment** Activity
- Review of Systems of Equations: Have each group solve one system using least 3 different methods. Discuss which method works best for each system.
- **Introductory Problem**
- Discussion: Point out the difference from the first problem. Discuss graphical representations.
- Discuss steps for solving a system of inequalities
- **Worksheet 1: Systems of Equations and Inequalities Practice**: Have students work together. To keep all students working, ask the least strong student at the table to explain method of solution.

DAY 3

- Linear Programming Definition
- Optimal Production Policy
- Feasible Set (region): Use *Example1* to facilitate the instruction.
- Have students use the vocabulary and anticipation guides.

HW: **7e** p. 167 #2, 3, 19
8e pp. 139-140 #4, 5, 21

DAYS 4 - 5

- Introduce the Corner Point Principle (build on *Example1*)
- **Worksheet 2: Corner Point Principle**

HW: 7e p. 168 #20-23
8e pp.139-140 #22-25

DAY 6

- **Branch and Bound** Activity. Show *Numb3rs* clip of "The Mole" from the Project TIME DVD. Give directions for solving a Linear Programming problem using graphing calculators. Then work on activity.

DAYS 7 - 8

- Mixture problems: Use *Example 2* to work on the concepts.
- **Worksheet 3: Mixture Problems**. Have students work in groups.

- **Linear Programming Additional Problems**. Assign each group two problems. There should be some redundancy here so that more than one group is working on each problem, yet all the problems are represented.

DAY 9

- Review for test: Assign each group a problem to work on in class. Have the groups present the solutions. An optional review is to use the following problems from the textbook:
  7e p.169 #31-34
  8e p.141 #33-36

DAYS 10-11

- Transportation problems: Begin with an example from the instructor's guide. Demonstrate the complexity of the problem and emphasize how using rows and columns help simplify the information in a usable way.
- Northwest Corner Rule
- Improving Feasible Solution
- Indicator Value
- **Worksheet 4: Northwest Corner; Transportation Problems**. Have students work in groups; report their solutions and/or draw solution on board.

HW: Read **7e** pp. 151-163
8e pp. 129-136

DAY 12

- Post-Assessment: A sample test is attached. The problems come from the FAPP Instructor Test Bank CD-ROM. The instructor may add more problems concerning the Northwest Corner Rule if appropriate.

INTRODUCTORY PROBLEM

These problems are meant to get students thinking about how to solve systems of equations and to motivate the idea of a feasible region for a linear programming problem.

The same questions without answers are copied on the next pages in a larger font so the instructor can copy them for handouts or onto an overhead transparency so that it isn't necessary to rewrite everything on the board.

PROBLEM 1

To receive messages, a spy must first send a pair of authorization numbers to his contact that satisfies the conditions of the two security agencies monitoring his movements. Agency A must receive the code and confirm that the sum of five times the first number plus three times the second number is eighty-four. Agency B must receive the code and confirm that four times the first number minus the second number is twenty-three. What two numbers should be sent when both agencies are monitoring his movements?

ANSWER

The solution of the linear system $5x+3y=84$ and $4x-y=23$ is $x=9$ and $y=13$.

***

The modified problem below gets at two ideas (1) having multiple possible answers, infinite in this case, and (2) writing inequalities and understanding the language of inequalities. The students are asked to represent their ideas graphically. Class discussion after the second problem might include a review of graphing.

PROBLEM 2

To receive messages, a spy must first send a pair of authorization numbers to his contact that satisfies the conditions of the two security agencies monitoring his movements. Agency A must receive the code and confirm that the sum of five times the first number plus three times the second number is less than or equal to eighty-four. Agency B must receive the code and confirm that four times the first number minus the second number is greater than or equal to twenty-three. What are some different ordered pairs of whole numbers can be sent when both agencies are monitoring his movements? Try to represent your solution graphically and shade the feasible region that contains these.

Answers will vary.

LINEAR PROGRAMMING
INTRODUCTORY PROBLEM 1

To receive messages, a spy must first send a pair of authorization numbers to his contact that satisfies the conditions of the two security agencies monitoring his movements. Agency *A* must receive the code and confirm that the sum of five times the first number plus three times the second number is eighty-four. Agency *B* must receive the code and confirm that four times the first number minus the second number is twenty-three. What two numbers should be sent when both agencies are monitoring his movements?

LINEAR PROGRAMMING
INTRODUCTORY PROBLEM 2

To receive messages, a spy must first send a pair of authorization numbers to his contact that satisfies the conditions of the two security agencies monitoring his movements. Agency A must receive the code and confirm that the sum of five times the first number plus three times the second number is less than or equal to eighty-four. Agency B must receive the code and confirm that four times the first number minus the second number is greater than or equal to twenty-three. What are some different ordered pairs of whole numbers can be sent when both agencies are monitoring his movements? Try to represent your solution graphically and shade the feasible region that contains these.

EXPLANATIONS OF WORKSHEETS

The following pages include a Pre-Assessment Activity, Worksheet #1, Classroom Example 1, Worksheet #2, Calculator Activity, Classroom Example 2, Worksheet #3, Additional Practice and Worksheet #4. Here are a few suggestions for using these materials. Since this is the first module that uses many algebra skills, students are encouraged to review the concepts of graphing linear inequalities, solving systems of equations and changing word problems into symbols.

The Pre-Assessment Activity is intended to give the instructor a snapshot of the students' algebra skills needed for this section. Give this assessment after the opening problem and discussions without any algebraic review.

SUGGESTED TIME: 15 minutes

Worksheet #1 gives the students practice solving systems of equations. In addition, students are encouraged to change sentences into equations after defining their variables. Emphasis on defining variables is important. The instructor should discuss the need for assigning appropriate labels and units on the axes for the graphs. Students are encouraged to work together on Worksheet #1.

SUGGESTED TIME: 35 minutes in groups
10 minutes share-out

Classroom Example #1: The instructor has the students discuss the first question in groups and report to the class the pros and cons of the jobs. The instructor should encourage discussion that businesses must also consider additional factors besides what the "math" solution demonstrates. For example, Kelly should consider items such as wear on the car when doing pizza deliveries or which job would be preferable on a resume. The instructor guides the students through the example while students fill in the sheet. Part (5) leads to the introduction of the Corner Point Principle.

SUGGESTED TIME: 25 minutes

Worksheet #2 gives more practice applying the Corner Point Principle. The students work in groups. The groups share their results for problems 5 and 6 with the class. The instructor discusses what other kinds of constraints might be applicable for the problems. Students discuss what people and/or businesses might use these kinds of problems.

SUGGESTED TIME: 40 minutes in groups
10 minutes share-out

The **Calculator Activity** is a lead-in to an activity based on the show NUMB3RS. The instructor uses the example with the directions to demonstrate how to use the TI-84+ when finding the feasible region and corner points. The instructor leads a discussion of why finding the intercepts and even the intersection points "by hand" might be useful before graphing (it helps to determine the correct viewing window!) The last page of this activity has extensions for the students.

SUGGESTED TIME: 10 minutes example
20 minutes of practice with problems from the instructor

Classroom Example 2 introduces the idea of a mixture problem. The instructor guides the students through the example emphasizing the layout of this example which guides students and organizes their work. The instructor introduces the use of a mixture chart as shown in the textbook. After the problem is completed (or while demonstrating it) review how to use the calculator to graph the information. There are many additional great examples to use on the Teacher Resource CD.

SUGGESTED TIME: 20 minutes

Worksheet #3 is intended for the students to work in groups. There are no guidelines given for format, but each group is expected to hand in a solution set that duplicates the Classroom Example demonstrated by the instructor. Have each group share one step of the solution. Discuss part (9) as a class.

SUGGESTED TIME: 15 – 20 minutes for students to work in groups
20 minutes for sharing-out

Additional Practice is intended to be a group activity and student teaching. Assign one or two problems to each group. Have the groups either make a poster of their solution or present it as a PowerPoint presentation. Remind the students that they are "the experts" for their assigned problem(s). Their presentation is meant to instruct the other students in the class.

SUGGESTED TIME: 90 minutes for solving the problem and creating a poster board
45 minutes on sharing-out

Worksheet #4 includes two practice problems using the Northwest Corner Rule. After the students work on the problems in groups, lead a discussion about what happens if the starting point is in a different corner. Also discuss the meaning of the indicator value.

SUGGESTED TIME: 15 minutes

PRE-ASSESSMENT ACTIVITY

Description and Answer Key

The activity on the next page is designed to be given to students at the very beginning of the chapter on Linear Programming. This activity will help you to assess how strong your students' algebra background is for the relevant skills they will need in this chapter. Students should work on this assessment individually at first, and then in groups so they can compare their answers and approaches.

Student should **not** use a calculator for this activity.

ESTIMATED TIME: 12 minutes

Solutions:

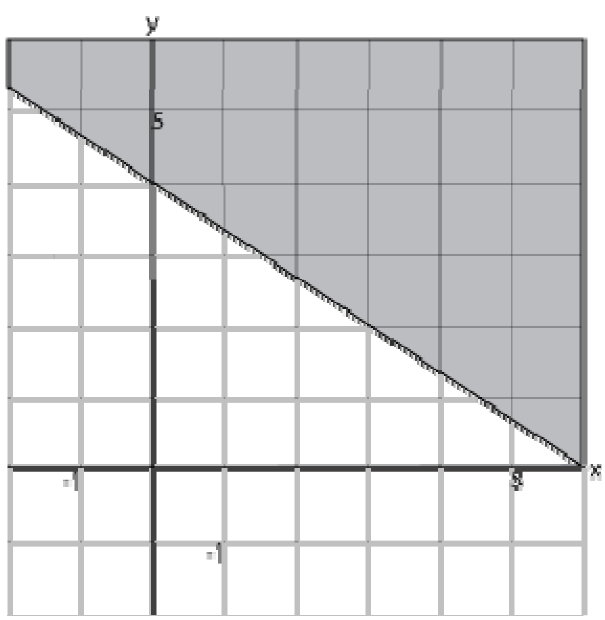
1) Solve the following system of equations. Be sure to show your work or explain how you found your solution.

$$x + y = 2$$
$$3x + 5y = 9$$

Solution: x=0.5, y=1.5

Students might solve this using either the addition method or substitution, or some combination of both. They may also try to solve the system graphically, in which case they should be especially careful to check that their estimated solution really works in the original system of equations.

2) Use the grid below to graph and shade the solution set for the inequality $2x + 3y \geq 12$.



The diagonal line that forms the bottom edge of the solution region is given by the equation 2x+3y=12.

The x-intercept of that line is x=12, and the y-intercept is y=4.

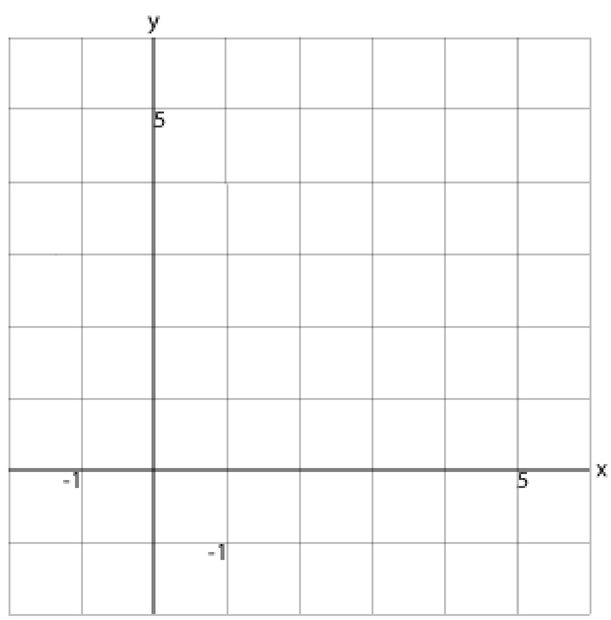
LINEAR PROGRAMMING
PRE-ASSESSMENT

1) Solve the following system of equations. Be sure to show your work to explain how you found your solution.

$$x + y = 2$$
$$3x + 5y = 9$$

2) Use the grid below to graph and shade the solution set for the inequality

$$2x + 3y \geq 12.$$



STUDENT HANDOUT: GRAPHING INEQUALITIES BY HAND

The handout on the next 4 pages is intended to be copied and given to students as a reference for graphing inequalities by hand. They should be allowed to use the reference while working on **Worksheet 1 - Systems of Equations and Inequalities** and on **Worksheet 2 - The Corner Point Principle**. By the time students get to the end of the module and are ready to take a test on this material, they should be able to graph inequalities without such a reference.

The instructor may wish to give students a few minutes of class time to read through the handout before giving them the first worksheet.

GRAPHING A SINGLE INEQUALITY

Step 1: Write the inequality in a form with the variable y isolated on one side.

Step 2: Draw the line that corresponds to the related equality (this is called the **boundary line**): draw the line dashed if the inequality is **strict** ($< or >$) and solid if the inequality is **not strict** ($\leq or \ \geq$).

Step 3: Shade the region above the line if the inequality is of the form $y \geq mx + b$ or $y > mx + b$. Shade the region below the line if the inequality is of the form $y \leq mx + b$ or $y < mx + b$.

Example 1: Graph the inequality $6x + 2y \leq 12$

Solution: Subtract $6x$ from both sides of the inequality to obtain

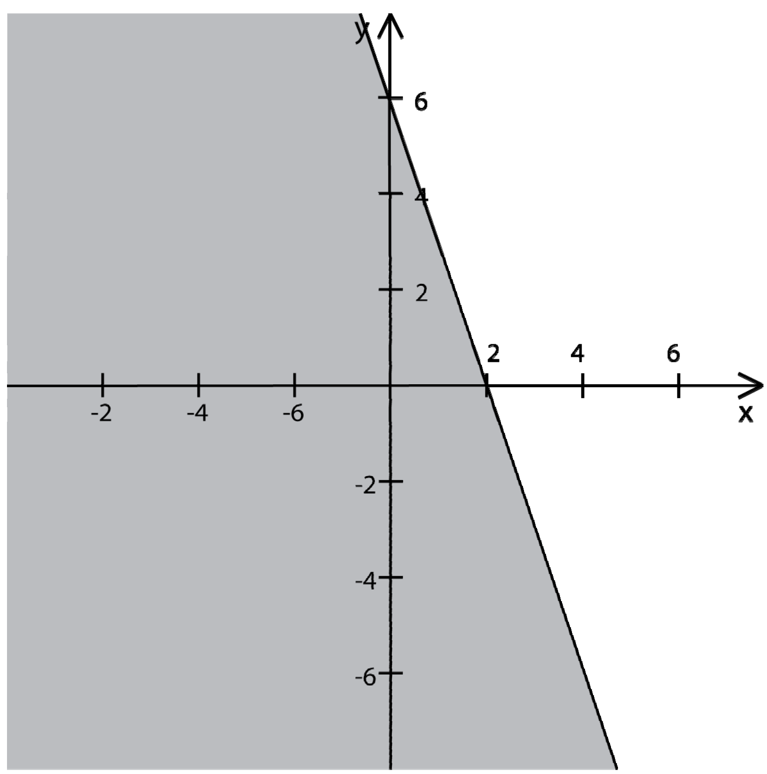
$$2y \leq -6x + 12$$

Then divide both sides by 2 to get

$$y \leq -3x + 6$$

(This has isolated y on one side of the inequality, as Step 1 suggests.)

The related equality for this is $y = -3x + 6$, and because the inequality is not strict, we will draw this boundary line as a solid line. Because the inequality is of the form $y \leq \cdots$, we will shade the region below this line.



Example 2: Graph the inequality $4x - 3y < 12$

Solution: Subtract $4x$ from both sides of the inequality to obtain

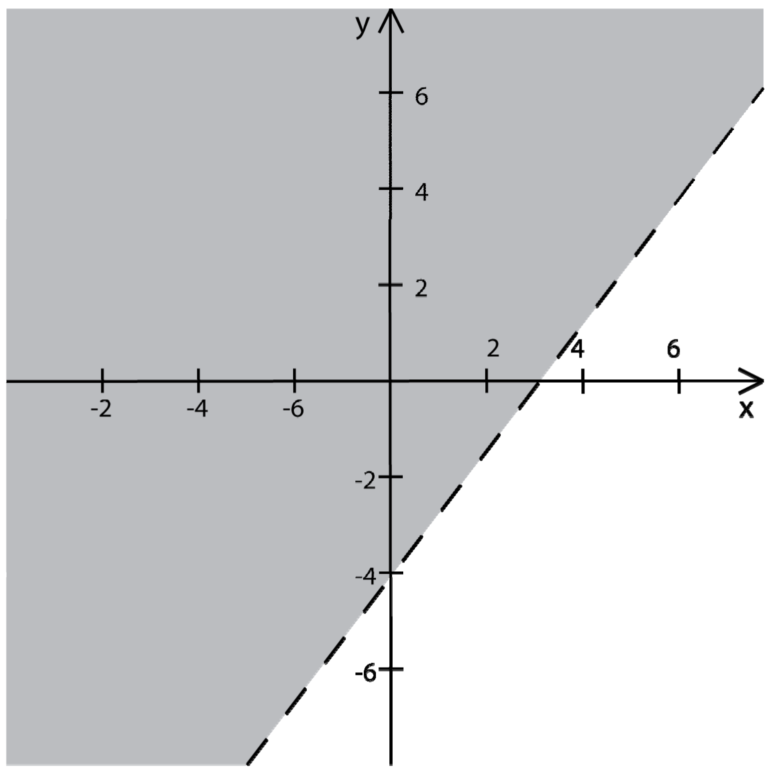
$$-3y < -4x + 12$$

Then divide both sides by -3 to get

$$y > \frac{4}{3}x - 4$$

(Recall that dividing by a negative number changes the direction of the inequality!)

The related equality for this is $y = \frac{4}{3}x - 4$, and because the inequality is strict, we will draw this as a dashed line. Because the inequality is of the form $y > \cdots$, we will shade the region above this line.



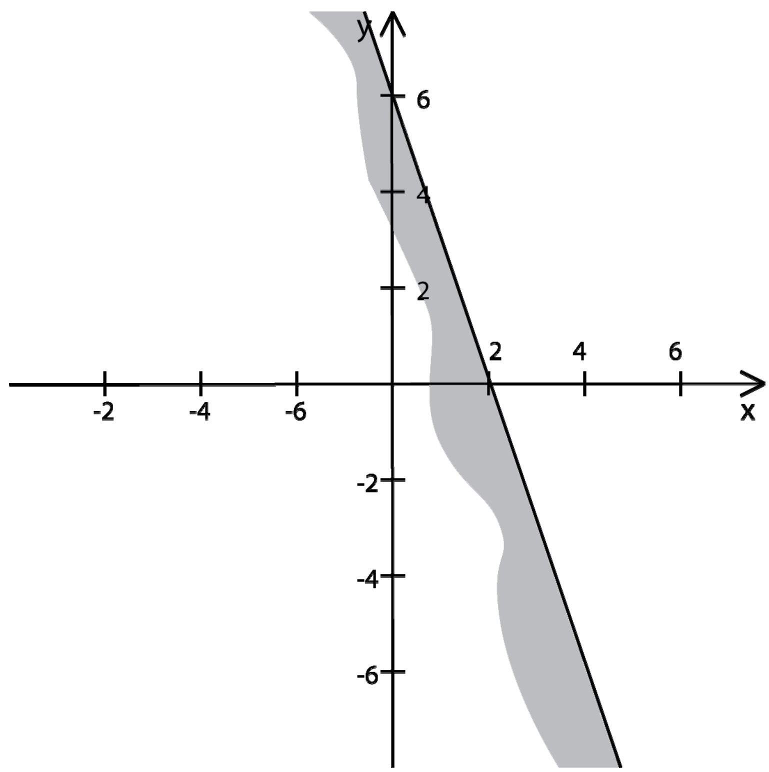
GRAPHING THE INTERSECTION OF SEVERAL INEQUALITIES

Next we consider the problem of graphically representing the set of points that simultaneously satisfy two or more inequalities.

Graph each of the inequalities on the same coordinate plane as illustrated in the previous examples, but do your shading very lightly so that you can see what region is shaded for all of the inequalities. It is recommended that you only shade near the boundary line for each inequality so that there is less to erase later.

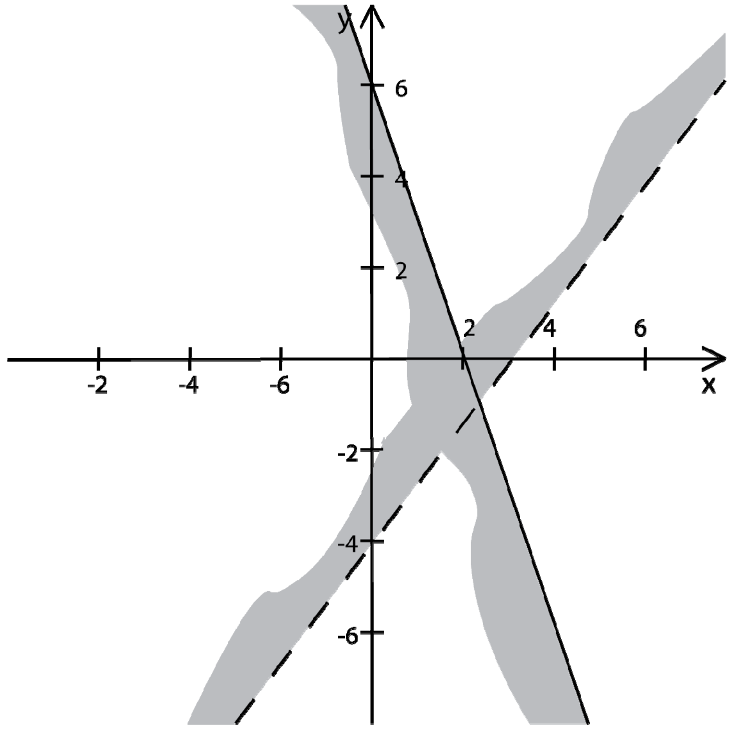
Example 3: Graph the common region for the inequalities $6x + 2y \leq 12$ and $4x - 3y < 12$

Solution: We begin by graphing the first inequality, just as we did in Example 1, but we only shade a little bit near the boundary line:

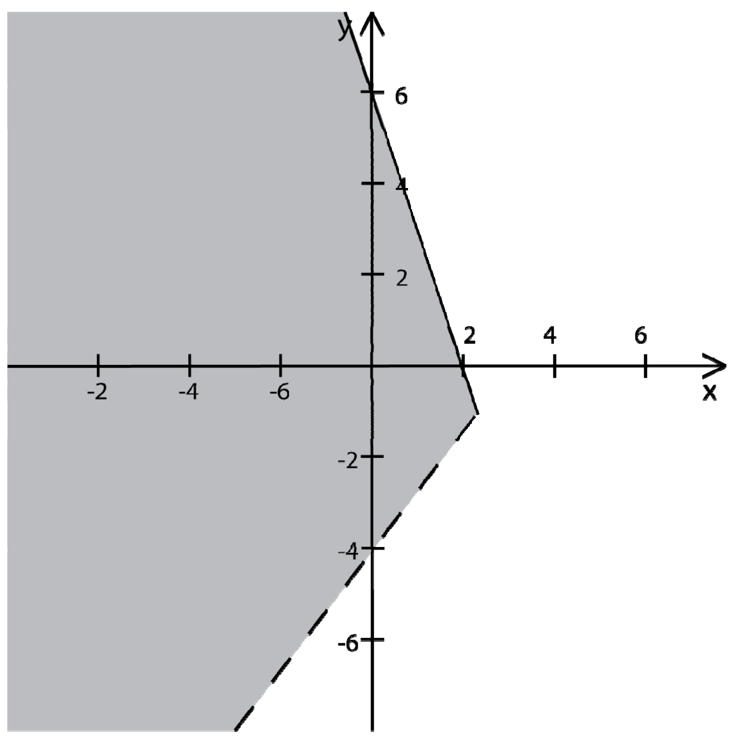


On the same coordinate axes, we now add the graph for the second inequality, $4x - 3y < 12$, again shading only part of the region near that boundary line:

Graphing Inequalities By Hand



From this, we can see that the region that gets shaded by both inequalities is the part of the coordinate plane on the left, so we shade in that entire region and erase the lines and shading that are outside of it:



This is the common region that represents all the points whose x and y coordinates satisfy both inequalities.

Linear Programming Worksheet #1:

Systems of Equations and Inequalities - Answer Key

FOR #1 – 3, SOLVE THE FOLLOWING SYSTEMS OF EQUATIONS:

1. $x = 6y + 18$ $3y + x = 36$	2. $3x + 5y = 7$ $x - 5y = 5$	3. $2x - 7y = 3$ $4x + y = 3$

Solutions:

x=30, y=2	x=3, y=-2/5	x=4/5, y=-1/5

FOR #4 – 7, DEFINE YOUR VARIABLES, CONVERT EACH SENTENCE INTO AN INEQUALITY, AND GRAPH THE CORRESPONDING REGION:

4. A daycare provides at least one supervisor for every three infants.

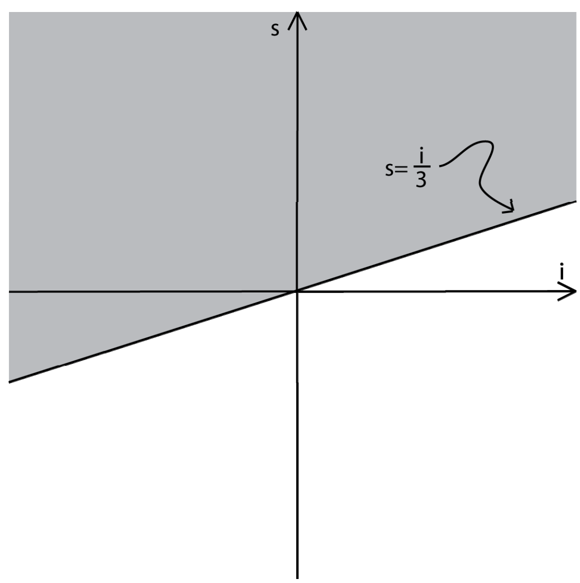
Variables:

S = # of supervisors

I = # of infants

Inequality:

s>=i/3



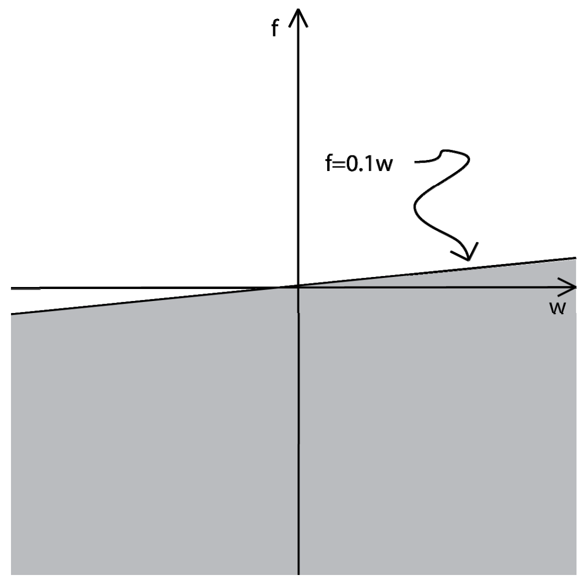
5. In lean beef, no more than 10% of the weight is fat.

Variables:

W = total weight of beef
F = weight of fat

Inequality:

f<=0.1w



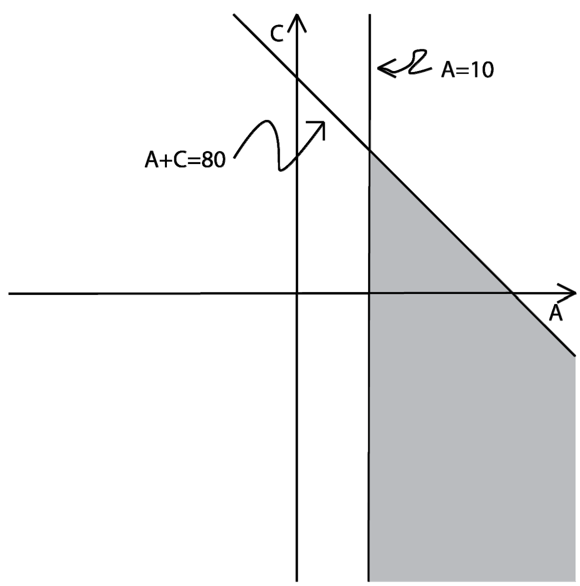
6. Children and adults tickets will be sold. A minimum of 10 adult tickets must be sold. No more than 80 total tickets can be sold.

Variables:

A = # of adult tickets sold
C = # of children tickets sold

Inequalities *(There are two!):*

A >= 10
A+C <= 80



For this problem, graph the region that is satisfied by both inequalities.

FOR #8 – 10, DEFINE YOUR VARIABLES AND CONVERT EACH SENTENCE INTO AN INEQUALITY:

Abdul owns a clothing store with the following conditions:

8. He wants to order up to 500 clothing items (shirts and pants) to display and sell for the fall.

Variables:

S = # of shirts
P = # of pants

Inequality:

S+P <= 500

9. He knows that he should have at least as many shirts as pants.

Inequality:

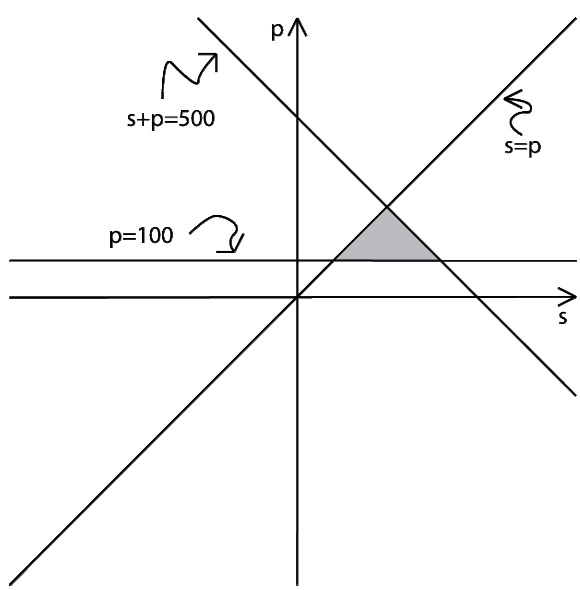
S >= P

10. He wants to have at least 100 pants.

Inequality:

P >= 100

Graph the region that satisfies all of the inequalities found above in questions 8-10. Be sure to label the axes appropriately:



Linear Programming Worksheet #1:

Systems of Equations and Inequalities

FOR #1 – 3, SOLVE THE FOLLOWING SYSTEMS OF EQUATIONS:

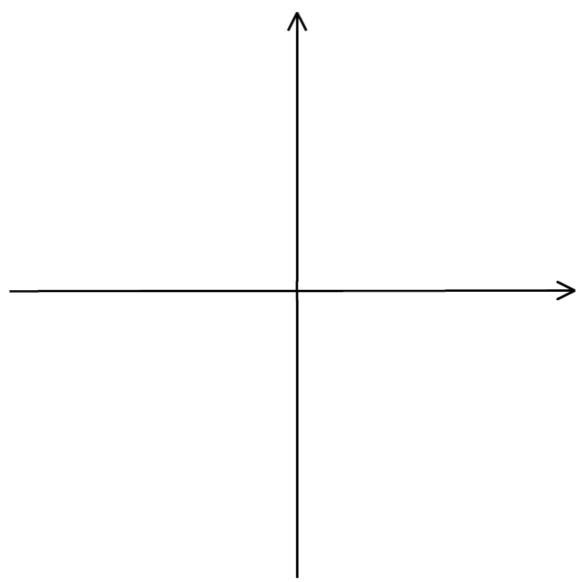
1.	$x = 6y + 18$ $3y + x = 36$	2.	$3x + 5y = 7$ $x - 5y = 5$	3.	$2x - 7y = 3$ $4x + y = 3$

FOR #4 – 6, DEFINE YOUR VARIABLES, CONVERT EACH SENTENCE INTO AN INEQUALITY, AND GRAPH THE CORRESPONDING REGION:

4. A daycare provides at least one supervisor for every three infants.

Variables:

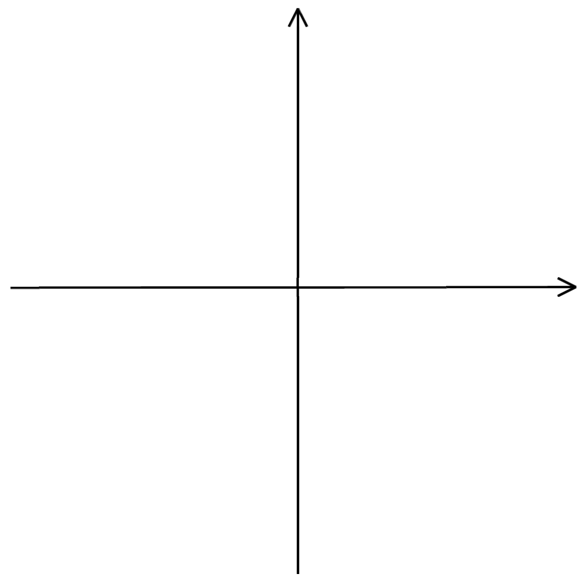
Inequality:



5. In lean beef, no more than 10% of the weight is fat.

Variables:

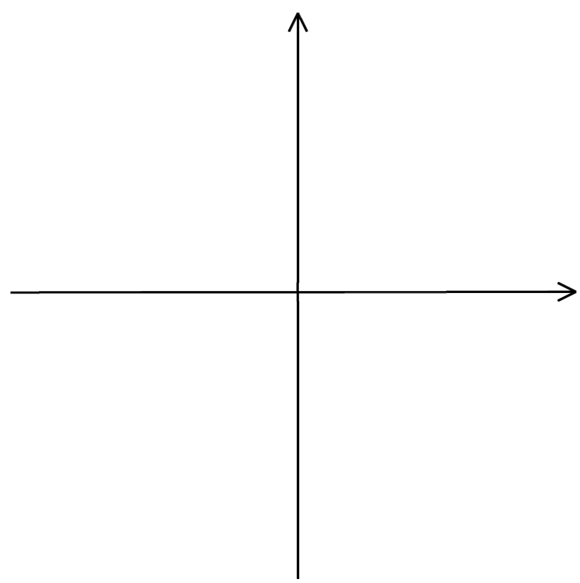
Inequality:



6. Children and adults tickets will be sold. A minimum of 10 adult tickets must be sold. No more than 80 total tickets can be sold.

Variables:

Inequalities *(There are two!):*



For this problem, graph the region that is satisfied by <u>both</u> inequalities.

FOR #7 – 9, DEFINE YOUR VARIABLES AND CONVERT EACH SENTENCE INTO AN INEQUALITY:

Abdul owns a clothing store with the following conditions:

7. He wants to order up to 500 clothing items (shirts and pants) to display and sell for the fall.

Variables:

Inequality:

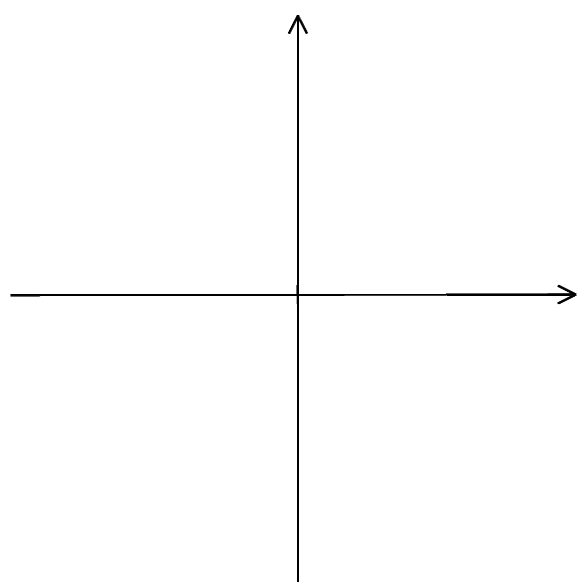
8. He knows that he should have at least as many shirts as pants.

Inequality:

9. He wants to have at least 100 pants.

Inequality:

Graph the region that satisfies <u>all</u> of the inequalities found above in questions 8-10. Be sure to label the axes appropriately:



4.1 Mixture Problems

Vocabulary Preview

Terms	??	?	!	Written Definition	Logograph
Linear programming					
Mixture problem					
Optimal production policy					
Mixture chart					
Minimum constraints					
Feasible Region					

Key: ?? I have NO idea what this means
? I have heard it before…but I'm not sure
! I know this word! It means…
Logograph: Sketch what your mind "sees" when you read each word.

ANTICIPATION GUIDE

INSTRUCTIONS:
Read each statement and write **Agree** in the blank if you believe the statement and could support it or put **Disagree** in the blank if you do not believe it or could not support it. After you finish reading the selection – we will revisit this and check the validity of each statement.

Before Reading		After Reading
	Linear programming is a management science technique that was formulated shortly after WWII.	
	Mixture problems only deal with food.	
	Linear programming helps maximize profits with the help of systems of inequalities.	

4.2 FINDING THE OPTIMAL PRODUCTION POLICY

Vocabulary Preview

Terms	??	?	!	Written Definition	Logograph
Corner point principle					
Optimal production policy					

Key: ?? I have NO idea what this means
? I have heard it before…but I'm not sure
! I know this word! It means…
Logograph: Sketch what your mind "sees" when you read each word.

ANTICIPATION GUIDE

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Before Reading		After Reading
	The transportation problem in linear programming is the same as the traveling salesman problem.	
	The Northwest corner rule was discovered in Seattle.	
	The stepping stone method helps improve the solution to a transportation problem when it is not optimal.	

4.3 WHY THE CORNER POINT PRINCIPLE WORKS

Vocabulary Preview

Terms	??	?	!	Written Definition	Logograph
Profit line					

Key: ?? I have NO idea what this means
? I have heard it before…but I'm not sure
! I know this word! It means…
Logograph: Sketch what your mind "sees" when you read each word.

ANTICIPATION GUIDE

INSTRUCTIONS:
Read each statement and write **Agree** in the blank if you believe the statement and could support it or put **Disagree** in the blank if you do not believe it or could not support it. After you finish reading the selection – we will revisit this and check the validity of each statement.

Before Reading		After Reading
	A **feasible region** has an infinite amount of points within.	
	The proof of the **corner point principle** deals with the geometric shape of the **feasible region.**	

4.5-4.6 TRANSPORTATION PROBLEMS

<u>Vocabulary Preview</u>

Terms	??	?	!	Written Definition	Logograph
Transportation problem					
Rim conditions					
Tableau					
Northwest corner rule					
Indicator value of a cell					
Stepping stone method					

Key: ?? I have NO idea what this means
? I have heard it before…but I'm not sure
! I know this word! It means…
Logograph: Sketch what your mind "sees" when you read each word.

ANTICIPATION GUIDE

INSTRUCTIONS:
Read each statement and write **Agree** in the blank if you believe the statement and could support it or put **Disagree** in the blank if you do not believe it or could not support it. After you finish reading the selection – we will revisit this and check the validity of each statement.

Before Reading		After Reading
	The **optimal production policy** is a point found anywhere in the feasible region.	
	The **corner point principle** states that the maximum value for the profit formula always corresponds to a corner point of the **feasible region**.	

CLASSROOM EXAMPLES 1 AND 2

The examples on the following pages are meant for the instructor to use for in-class demonstration. They are formatted so that you can copy these onto transparencies to use with an overhead projector; you could also make copies to give out to students if you wish.

Example 1 is meant to be used on days 3-4 of the Linear Programming module. It illustrates the essential steps of solving a mixture problem graphically from a written description: assigning the variables, writing the constraint inequalities, identifying the objective function, graphing the feasible region, testing the corner points, and locating the optimum value. Additionally, it is written in an open-ended format that allows the teacher to follow the students' lead in answering either one of two questions:

- What is the fewest number of hours Kelly needs to work to earn enough money; or
- What is the maximum amount of money that Kelly can earn?

It is recommended that the instructor actually answer both of these questions, one at a time, so that you will actually have demonstrated two examples of these problems. You can follow the students' lead on which question to answer first. Ideally, they will suggest what the question should be on their own.

Maximizing Kelly's earnings is actually a slightly simpler problem, because it has one fewer constraint.

Example 2 is meant for day 8 of the module. It is another example of a mixture problem, and it is meant chiefly as review. The previous few classes should have had students working with calculators, not specifically working on linear programming problems, so this gives them an opportunity to recall the process. Also, at this stage it will be possible for them to check their work using the graphing calculators.

EXAMPLE 2: ANSWER KEY

1. x= Motorcycle cover
   y = Boat cover

2. $2x + 4y \leq 100$
   $y \geq 2x$
   $x \geq 0$
   $y > 0$

3. Maximize $9x + 15y$

4. Graph

5. Corner Points

x-variable	y-variable	Profit
0	0	0
0	25	375
10	20	390

6. Kevin should make 10 motorcycle covers and 20 boat covers.

Example 1: Kelly is a student who lives close to campus. She can earn $10 per hour delivering pizza and $7.50 per hour working in the campus computer lab. She can work up to 30 hours per week with the class load that she is carrying, and she must earn $240 to cover all of her bills.

Describe the problem you are going to solve (maximize or minimize something):

1. Assign the variables:

2. Write (and label) the constraint inequalities:

Example 2: Kevin has 100 yd^2 of fabric and unlimited thread. 1 motorcycle cover requires 2 yd^2 of fabric and brings in a profit of $9. 1 boat cover requires 4 yd^2 and brings in a profit of $15. Demand requires at least twice as many boat covers as motorcycle covers. If he wants to maximize his profit, how much of each item should he make?

1. Assign the variables:

2. Write the constraint inequalities:

3. Objective function:

LINEAR PROGRAMMING WORKSHEET #2
ANSWER KEY

1. b) Corner points: (0,0)
   (0,4)
   (6, 0)
   (9.23, 2.153)

2. $5x - 3y$
   (1,1) = 2
   (2,1) = 7
   (3,2) = 9 maximizes profit
   (2,4) = -2
   (1,5) = -10

3. *C* is not a corner point

4. $2x + (1/2)y \leq 40$

   $(1/2)x + (1/4)y \leq 2$
5. $12x + 10y \leq 640$
   $4x + 3y \leq 480$

6.

X	*Y*	profit
3	2	13
3	6	21
8	2	28
6	4	26

7. Maximum profit of $28 occurs at (8,2)

LINEAR PROGRAMMING WORKSHEET #2

The Corner Point Principle

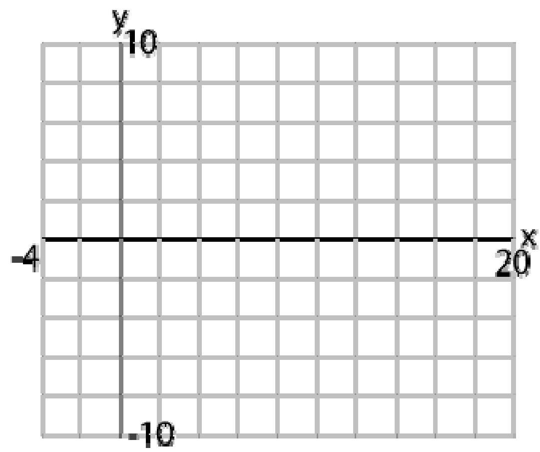
1. a) Graph the feasible region for the following inequalities:

$2x-3y\leq12$

$x+5y\leq20$

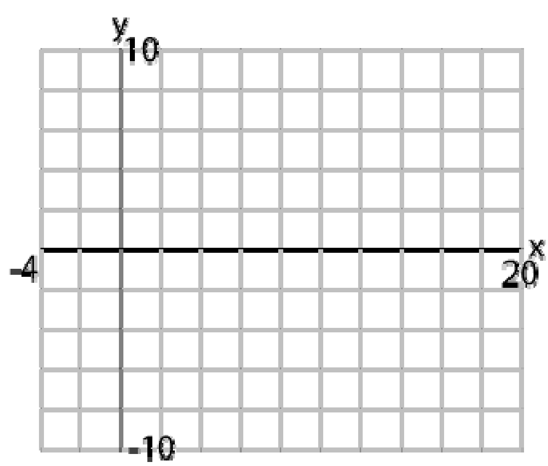
$x\geq0$

$y\geq0$



b) Find the corner points for your answer to #1. Verify your answers using algebra.

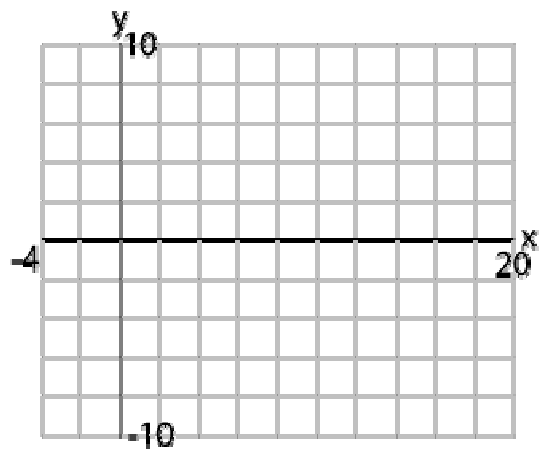
2. Suppose a feasible region has five corners, at points (1,1), (2,1), (3,2), (2,4), (1,5), and the profit formula in dollars is $P=5x-3y$. Find the point that maximizes the profit. Show your work



3. Consider the feasible region identified by the inequalities $x \geq 0$, $y \geq 0$, $3x+y \leq 10$, $x+2y \leq 5$

Which point is not a corner of the region?

a. (0,2.5)
b. (3,1)
c. (5,0)



4. For the description, write appropriate resource-constraint inequalities. Let x be the number of large trees and y the number of small trees. Maintaining a large tree takes 2 hours of pruning time and 30 minutes of shredder time: maintaining a small tree takes 30 minutes of pruning time and 15 minutes of shredder time. There are 40 hours of pruning time and 2 hours of shredding time available.

5. For the description, write appropriate resource-constraint inequalities. Let x be the number of salami and y be the number of bologna. Manufacturing one salami requires 12 oz of beef and 4 oz of pork. Manufacturing one bologna requires 10 oz of beef and 3 oz of pork. There are 40 lb of beef and 480 oz of pork available.

6. Find the maximum value of P where $P=3x+2y$ subject to the constraints

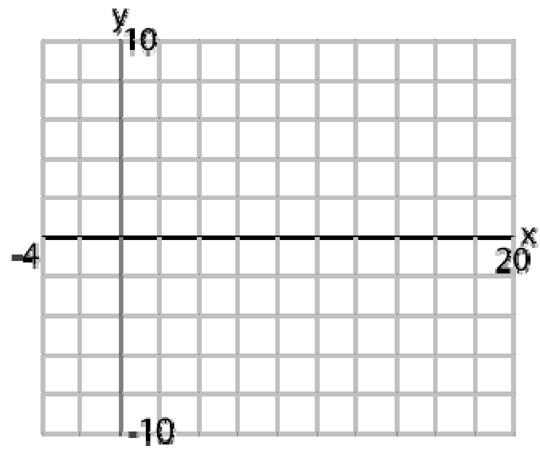
$x \geq 3$, $y \geq 2$, $x+y \leq 10$, $2x+3y \leq 24$

a) Graph the feasible regions using:

Intercepts

Lines

Shading



b) Find the corner points using algebra and set up the profit table.

X	**Y**	**Profit**

c) What is the maximum profit and which point(s) give the maximum?

NUMB3RS Activity: Branch and Bound
Episode: "The Mole"

Topic: Linear Programming **Grade Level:** 9 - 12

Objective: Forcing integral solutions

Time: 20 minutes

Materials: TI-83 Plus/TI-84 Plus graphing calculator with Inequality Graphing App installed

Introduction

Charlie and Amita are trying to determine where an FBI mole will make the next information drop. Charlie proposes that "by running an algorithm that analyzes the features of the existing meeting places to find key variables, then applying a branching and bounding algorithm, I can hopefully give you a location for the mole's next probable destination."

Charlie is talking about a program to optimize the likelihood of an outcome. One method of optimization often studied in high school math courses is linear programming. The branch and bound algorithm Charlie mentions can be used when integer solutions are desired, but not attained.

This activity assumes that students already understand the ideas behind linear programming. Rather than introduce linear programming, this activity is meant to provide an area for further study and to discuss using the calculator to find the feasible region and points of intersection for a system of inequalities.

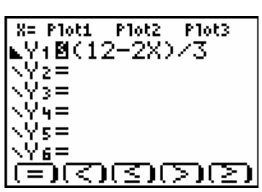
Discuss with Students

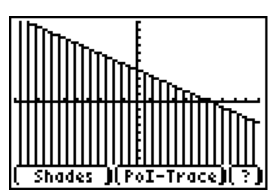
Consider the system below.

$2x + 3y \leq 12$
$x - y \leq 4$
$y < 3$
$x, y \geq 0$

To enter the inequalities into the calculator, each one must be solved for *y*. Rewrite the first two constraints into calculator ready form.

To begin, turn on the Inequality Graphing feature by pressing the [APPS] and choosing **Inequalz**. Press the [Y=] key to enter the first inequality. When the cursor is placed on the equal sign, inequality options will appear at the bottom of the screen. To enter the first inequality, select ≤ ("less than or equal to") by pressing [ALPHA] [F3] (the [ZOOM] key) and enter the inequality. To view the graph, press [ZOOM] key and select **6:ZStandard**. The Y= and graph screens are shown below.





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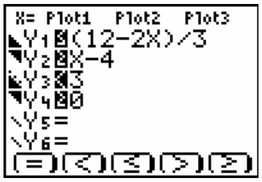
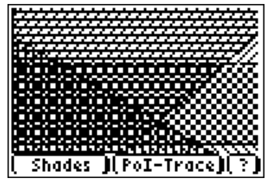
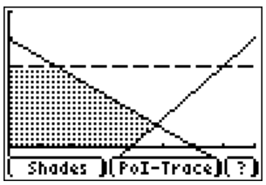
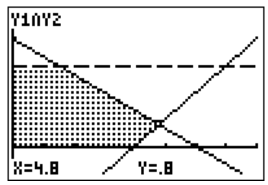
Enter the other three inequalities written in terms of *y*. The remaining condition will be entered by limiting the viewing window – set the window to *x*[0, 8] and *y*[–1, 5], then press [GRAPH]. The Y= and graph screens are shown below.





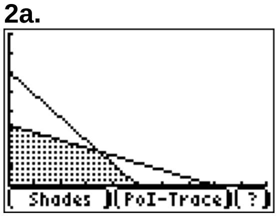
The screen is full of shadings that are difficult to distinguish from each other. Press [ALPHA] [F1] and choose **1: Ineq Intersection** to see the intersection of the shaded regions (as shown below). Press [ALPHA] [F4] to find the points of intersection, which are (4.8, 8), (1.5, 3), and (4, 0). Note that the two corner points on the *y*-axis will not be indicated because the constraint *x* ≥ 0 was never entered.



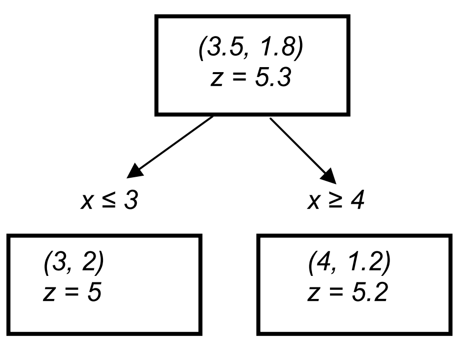


Discuss With Students Answers:
1. *y ≤ (12 – 2x)/3; y ≥ x – 4*

Student Page Answers:
1. *Maximize z = x + y; Constraints: 2x + 5y ≤ 16, 6x + 5y ≤ 30; x, y ≥ 0*
2a.



2b. *(0, 0), (5, 0) (3.5, 1.8), (0, 3.2)* **2c.** *optimal solution is z = 5.3 when x = 3.5 and y = 1.8* **3.** *The values of x and y are not integers.* **4.** *z = 31/6; x = 25/6, y = 1* **5.** *z = 5, x = 3, y = 2* **6.** *z = 5 See the chart below.*



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EXTENSIONS

When more than two variables are used in a linear programming problem, the graphical analysis becomes more challenging. For these cases, an approach called the Simplex Method can be used. Investigate this method, starting with the website below.

http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/tutorialsf4/frames4_3.html

ADDITIONAL RESOURCES

- The website below shows examples of the simplex method and dual simplex algorithm. http://www.egwald.com/operationsresearch/orpage.php3
- The website below uses Linear Programming with Major League Baseball. http://riot.ieor.berkeley.edu/~baseball/
- The website below has a Flash Application of the simplex method as well as an add-in to Excel that provides instruction for three different algorithms for solving linear programming. The first two are simplex methods that traverse the boundary of the feasible polytope and the last is an interior point method. http://www.me.utexas.edu/~jensen/ORMM/methods/unit/linear/index.html
- The Inequality Graphing App comes preloaded on the TI-84 Plus and TI-84 Plus Silver Edition graphing calculators. For the TI-83 Plus Silver Edition, this App can be downloaded for free from http://education.ti.com/inequality.

Name ____ Date _______________

NUMB3RS Activity: Branch and Bound

Charlie and Amita are trying to determine where an FBI mole will make the next information drop. Charlie proposes that "by running an algorithm that analyzes the features of the existing meeting places to find key variables, then applying a branching and bounding algorithm, I can hopefully give you a location for the mole's next probable destination."

Charlie is talking about a program to optimize the likelihood of an outcome. One method of optimization often studied in high school math courses is linear programming. The branch and bound algorithm Charlie mentions can be used when integer solutions are desired, but not attained.

Consider the following hypothetical situation: Charlie is breaking codes for the FBI using two different methods: *brute force* and *linear cryptanalysis*. To break a code with the first method, Charlie needs 2 hours of analysis and 6 hours of runtime on the computer. The second method requires 5 hours of analysis and 5 hours of runtime on the computer. If he has 16 hours of analysis and 30 computer hours available, what is the greatest number of codes that Charlie can break?

1. Write the goal and constraints for the problem. Let x represent the number of codes broken with brute force and let y represent the number of cases broken with linear cryptanalysis.

To solve a linear programming problem, first graph the system of inequalities given by the constraints. The resulting area is called the feasible region and the corner points of the region are the potential optimal values. The goal of the problem is to either maximize or minimize an expression; the corner point with the coordinates that satisfy that goal provides the optimal solution.

2. a. Use the Inequality Graphing App on your calculator to graph the feasible region. Set the viewing window to x[0, 10] and y[–1, 8].

b. Identify the corner points of the feasible region.

c. Evaluate the goal expression for each corner point and determine the optimal solution.

3. Partially breaking a code is not a desirable result. What is wrong with your solution to Question 2c?

The branch and bound algorithm will allow us to force the variables to be integers, while still optimizing the situation. The first step is to *branch* on a variable that is not an integer, creating two new problems. For simplicity with the calculator, we will choose to branch on y. The current solution is $y = 1.8$, so one branch adds the constraint $y \leq 1$ to the original problem and the other branch adds $y \geq 2$.

4. Solve the system again with the added constraint $y \leq 1$.

5. Solve the system with the added constraint $y \geq 2$.

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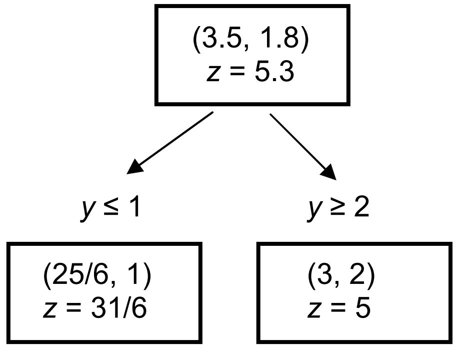
The solutions are shown in the chart below.



The right branch is complete, since the solution is integral. The left branch appears to require more branching. However, rounding down shows that the solution to this branch will be no better than 5. We previously determined an integral solution with that value from the right branch; so the left branch is *bounded* by that value and our optimal solution is 5. This means that Charlie can break at most 5 codes.

6. Repeat the branch and bound algorithm branching on the x variable. Make a chart to show progress.

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LINEAR PROGRAMMING WORKSHEET #3
ANSWER KEY

1.

	Material (yd)	Profit
Shirt	3	5
Vest	2	2
Total	600	

2. Shirt = x
   Vest = y

3. Profit: $5x + 2y$

4. $x \geq 100$, $y \geq 30$, $3x + 2y \leq 600$

5. Corner Points: (100,30), (180, 30), and (100, 150)

6. Profit (100,30) = 560
   Profit (180, 30) = 960
   Profit (100, 150) = 800

8. 180 shirts and 30 vests

9. With $x \geq 0$ and $y \geq 0$, new corner points are: (0, 300), (0,0) and (200, 0). Maximum profit is now $1500 by making 300 shirts and no vests.

LINEAR PROGRAMMING WORKSHEET #3

MIXTURE PROBLEMS

FOR #1-7 USE THE FOLLOWING:

A clothing manufacturer has 600 yd of cloth available to make shirts and decorated vests. Each shirt requires 3 yd of material and provides a profit of $5. Each vest requires 2 yd of material and provides a profit of $2. The manufacturer wants to guarantee that under all circumstances there are minimums of 100 shirts and 30 vests produced.

1. Make a mixture chart for the problem.

2. Label your variables.

3. Using the mixture chart, write the profit formula.

4. Write the resource and minimum constraint inequalities.

5. Draw the feasible region for those constraints and find the coordinates of the corner points using algebra.

6. Evaluate to determine the production policy that best answers the question.

7. Check your work by using the graphing calculator.

8. How many of each garment should be made to maximize profit?

9. If there are no minimum quantities, how, if at all, does the optimal production policy change?

LINEAR PROGRAMMING ADDITIONAL PRACTICE

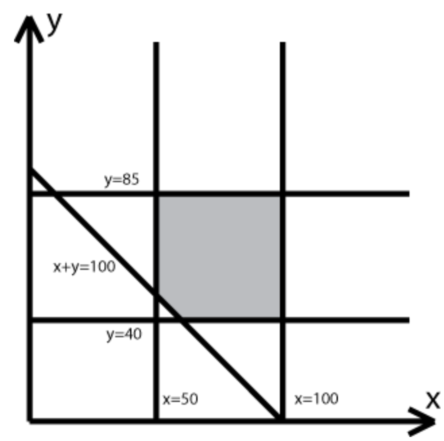
ANSWER KEY

1. Let x = the number of ski helmets produced
   y = the number of bicycle helmets produced

Constraints: $50 \le x \le 100$
$40 \le y \le 85$
$100 \le x+y$

Objective function: (maximize) Revenue = 5y – 2x

The feasibility region graphs:



x-values	y-values	Revenue
50	80	**$380**
50	50	**$150**
100	40	**$0**
60	40	**$80**
100	80	**$200**

To maximize revenue the company should produce 50 ski helmets and 80 bicycle helmets.

2. Let x = the number of #9310 cabinets purchased
   y = the number of #9218 cabinets purchased

Constraints: $0 \le x$
$0 \le y$
$45x + 90y \le 630$
$4.5x + 6y \le 54$

Objective function: (maximize) Volume = 13.5x + 20.25y

The feasibility region graphs:

x-values	y-values	Volume (ft^3)
0	0	**0**
0	7	**141.75**
12	0	**162**
8	3	**168.75**

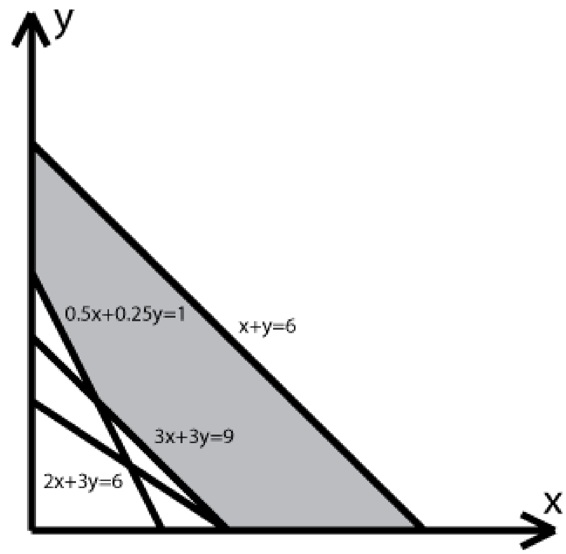
To maximize volume you should purchase 8 #9310 cabinets and 3 #9318 cabinets.

3. Let x = number of ounces of Science Diet used
   y = number of ounces of Formula Plus used

Constraints: $6 \leq 2x + 3y$
$1 \leq 0.5x + 0.25y$
$9 \leq 3x + 3y$
$x + y \leq 6$

Objective function: (minimize) Cost = $0.05x + 0.08y$

The feasibility region graphs:



x-values	y-values	Cost (in dollars)
3	0	**$0.15**
6	0	**$0.30**
0	4	**$0.32**
0	6	**$0.48**
1	2	**$0.21**

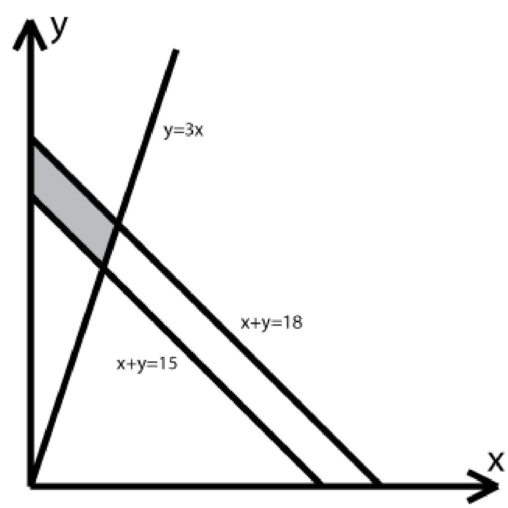
To minimize cost use 3 ounces of Science Diet only.

4. Let x = the amount invested in municipal bonds (in thousands of dollars)
   y = the amount invested in CD's (in thousands of dollars)
   18-x-y = the amount invested in high risk funds (in thousands of dollars)

Constraints: $0 \leq x$
$0 \leq y$
$15 \leq x+y \leq 18$
$3x \leq y$

Objective function: (maximize) Revenue = $1.62 – 0.0374x – 0.0675y$

The feasibility region graphs:



x-values	y-values	Revenue (in dollars)
0	15	**$607.50**
0	18	**$405**
4.5	13.5	**$720.38**
3.75	11.25	**$80**

To maximize revenue you need to invest \$3,750 in municipal bonds, \$11,250 in CD's and \$3000 in high risk funds.

5. This problem has four variables. We need to reduce these to two in order to graph on the xy-coordinate system.

Let A_s = the number of trusses to be delivered to AC from the southern warehouse.
A_n = the number of trusses to be delivered to AC from the northern warehouse.
D_s = the number of trusses to be delivered to DC from the southern warehouse.
D_n = the number of trusses to be delivered to DC from the northern warehouse.

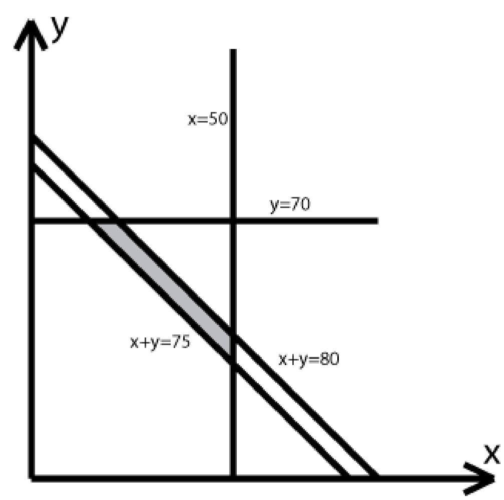
Since $A_s + A_n = 50$ we have $A_n = 50 - A_s$
Since $D_s + D_n = 70$ we have $D_n = 70 - D_s$

Constraints: $0 \leq A_s \leq 50$
$0 \leq D_s \leq 70$
$0 \leq A_s + D_s \leq 80$
$0 \leq A_n + D_n \leq 45$ with substitution this gives $75 \leq A_s + D_s \leq 120$ but the constraint above means that you really have $75 \leq A_s + D_s \leq 80$

Objective function: (minimize) Shipping Cost = $234 + 0.4A_s + 0.2D_s$

The feasibility region graphs:

A_s-values	D_s-values	Cost (in dollars)
50	30	**\$260**
50	25	**\$259**
10	70	**\$252**
5	70	**\$250**



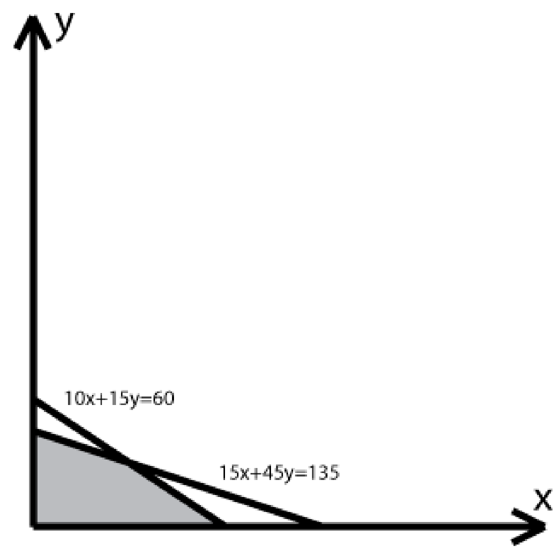
To minimize cost Metal Pro will ship5 trusses from the southern warehouse to AC, 45 trusses from the northern warehouse to AC, 70 trusses from the southern warehouse to DC and no trusses from the northern warehouse to DC.

6. Let x = the number of small garages Pedro will build
   y = the number of large garages Pedro will build

Constraints: $0 \leq x$
$0 \leq y$
$10x + 15y \leq 60$
$15x + 45y \leq 135$

Objective function: (maximize) Profit = 390x + 520y

The feasibility region graphs:



x-values	y-values	Profit (in dollars)
0	3	**$1,560**
3	2	**$2,210**
6	0	**$2,340**

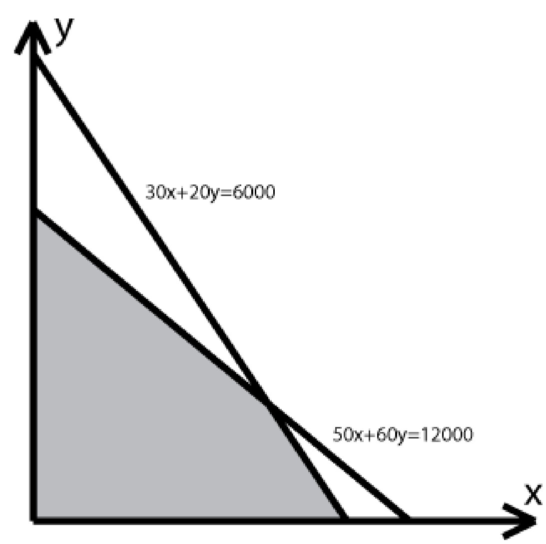
To maximize profit, Pedro should sell 6 small garages and not make any large garages.

7. Let x = number of systems produced in Federal Way.
   y = number of systems produced in Auburn.

Constraints: $30x + 20y \leq 6000$
$50x + 60y \leq 12000$
$0 \leq x$
$0 \leq y$

Objective function: (maximize) Production = x + y

The feasibility region graphs:



x-values	y-values	Production (number of systems)
0	200	**200**
0	0	**0**
200	0	**200**
150	75	**225**

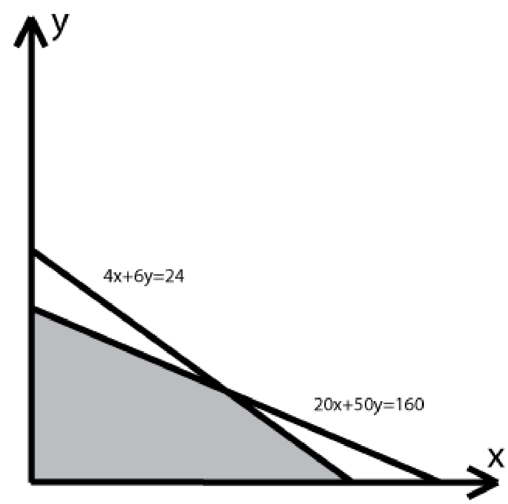
To maximize production, the company needs to produce 150 in the Federal Way store and 75 in the Auburn store.

8. Let x = the number of 24-passenger buses
   y = the number of 56-passenger buses

Constraints: $20x + 50y \leq 160$ (for students only so don't count the chaperones needed.)
$4x + 6y \leq 24$ (for the chaperones only.)
$0 \leq x$
$0 \leq y$

Objective function: (minimize) Cost = 1520x + 2290 y

The feasibility region graphs:



x-values	y-values	Cost
0	16/5	$7328 * actually this is not correct since you can't order partial buses so the actual cost is **$9160**
8	0	**$12160**
3	2	**$9140**

So it is cheapest to get 3 24 passenger buses and 2 56 passenger buses. Ask the students how many chaperones will be coming. Also have students address the last question about vehicles that come equipped with restrooms.

LINEAR PROGRAMMING – ADDITIONAL PRACTICE

For each question below, start by identifying and writing down the objective function and the constraint inequalities. Draw a careful graph of the feasible region and solve for the corner points algebraically. Then use the objective function to determine the optimal choices.

When answering the questions, write down at least one additional consideration that might impact your final answer. Remember to think in practical terms!

1. A small company, the Creative Cover, produces top of the line helmets for skiing and bicycling. Long range projections indicate an expected demand of at least 50 ski helmets and 40 bicycle helmets every day. Since the company is so small, no more than 100 ski helmets and 85 bicycle helmets can be produced on a daily basis. Creative Cover has a shipping contract that requires a minimum of 100 helmets to be shipped each day.

   If each bicycle helmet results in a \$5 profit but each ski helmet sold gives a \$2 loss, how many helmets of each type must be produced each day to maximize net profits?

2. Your new job as an office manager for a start-up company requires that you purchase storage cabinets for the office. The office has room for no more than 54 square feet of cabinets. After a bit of research at Office Max, you find out that one cabinet, model #9310 costs \$45 per unit, requires 4.5 square feet of floor space, and holds 13.5 cubic feet of storage. The second cabinet, model # 9318, costs \$90 per unit, requires 6 square feet of floor space, and holds 20.25 cubic feet of storage. The boss has given you \$630 for the purchase, though you don't have to spend that amount (you can spend less!) How many of which model should you buty in order to maximize the storage volume?

3. At Petsto, the white mice need a diet to ensure optimal health before they are sold. The research shows that the mice need a daily diet with a minimum of 6 grams (g) of fat, 1 g of protein and 9 g of carbohydrates. The mice may not be fed more than 6 ounces of food a day.

   The custom-blended food is very expensive, so Petsto orders two food supplies and makes a blend for an optimum mix of food. The Science Diet contains 2 g of fat, 0.5 g of protein and 3 g of carbohydrates per ounce. It costs $0.05 per ounce. Formula Plus contains 3 g of fat, 0.25 g of protein and 3 g of carbohydrates and costs $0.08 per ounce.

   What is the optimal blend?

4. Your great aunt left you a trust fund of $18,000 that you may now have to invest with some restrictions. You have three different funds from which to choose. The municipal bond fund has a 5.26% return, U.S. Bank has certificate of deposits (CDs) with a 2.25% return and the high-risk fund has an expected (but not guaranteed) return of 9%. To minimize risk, you decide to invest a maximum of $3,000 in the high-risk account. A stipulation of the trust indicates that you must invest at least three times as much in the bank CDs as in the municipal bonds. Assuming the year-end returns are as expected, what are your best investment choices to maximize your return? Be sure to state how much you will invest in each fund.

5. Metal Pro has two ware house locations in Seattle. The main office receives orders from two customers, Armstrong Construction (AC) and Delhi Construction (DC), each requiring 10 foot steel trusses for construction. AC needs 50 trusses and DC requires 70 trusses.

   The warehouse in south Seattle has 80 trusses in stock; the north Seattle warehouse has 45 trusses in stock. Metal Pro's delivery costs vary depending on the distance from its warehouses to the construction sites and the number of trusses delivered. The cost to deliver to the AC site is $2.00 per truss from the southern warehouse and $1.60 per truss from the northern warehouse. The cost to deliver to the DC site is $2.40 per truss from the southern warehouse and $2.20 from the northern warehouse.

   Determine the best delivery arrangement that minimizes the cost of delivery for Metal Pro.

6. Pedro builds garages. He uses 10 sheets of dry wall and 15 studs for a small garage and 15 sheets of dry wall and 45 studs for a larger garage. Currently he has 60 sheets of dry wall and 135 studs available. If Pedro makes $390 profit on a small garage and $520 on a larger garage, how many of each type of building should Pedro build to maximize his profit?

7. An electronics company builds stereo systems in two different factories. At the Federal Way factory, 30 hours are required to produce one system. The Auburn factory takes 20 hours. To produce one system, the costs of producing each system are $50 in Federal Way and $60 in Auburn. The company's labor force can provide 6000 hours of labor each week and resources are $12,000 each week. How should the company allocate its labor and resources to maximize the number of products produced?

8. The Renton School District is sponsoring an overnight trip for graduating seniors; BBgetaway coaches is offering the following rates:

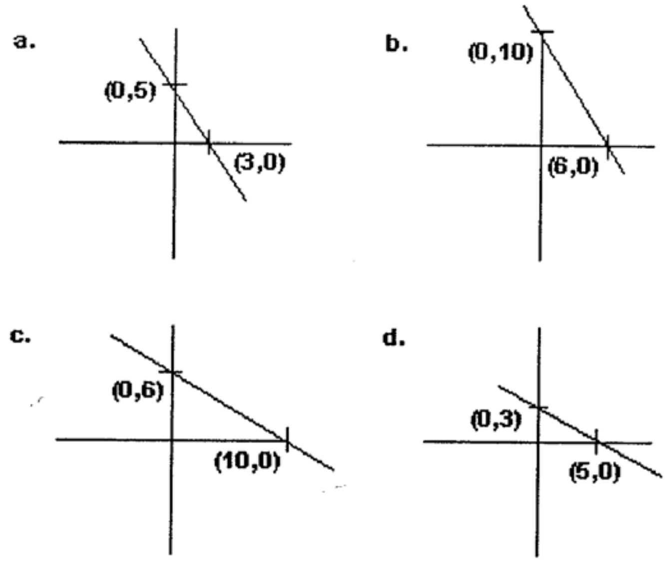
Vehicle Capacity	**Overnight Charter Rate**
24	$1520
56	$2290

The district anticipates at most 160 students will attend. The 24-passenger bus requires 4 chaperones. The 56-passenger bus requires 6 chaperones. At most 24 chaperones are available to go on the trip. How many of each type of vehicle should the district hire to minimize the transportation costs?

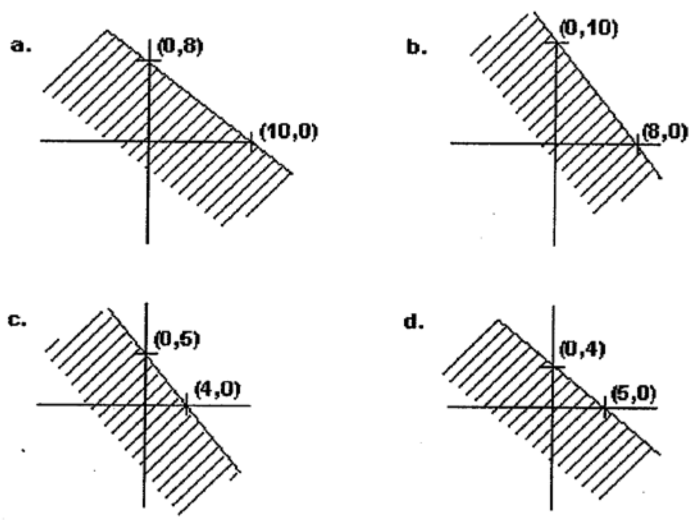
Would your recommendation change if you knew that the 56-passenger vehicles come equipped with restrooms?

LINEAR PROGRAMMING REVIEW **NAME** ___________________

1. Find the graph of the equation 3x + 5y = 30



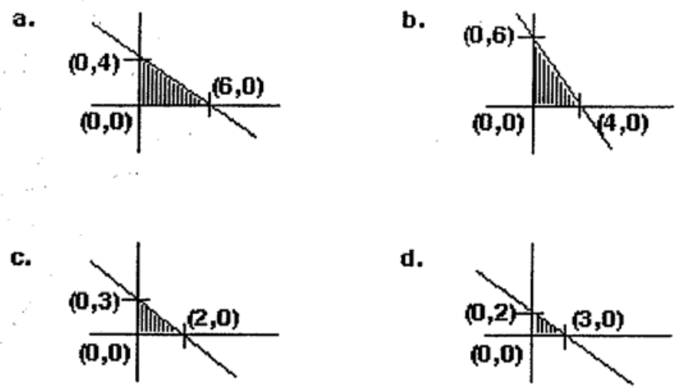
2. Find the graph of the inequality $4x + 5y \le 40$.



3. Find the point of intersection of the lines whose equations are given by 2x + 3y = 12 and x + 5y = 13.
   a. (2,3) b. (3,2) c. (6,0) d. (-2,3)

4. For the constraint inequalities given below, determine which feasible region is correct.

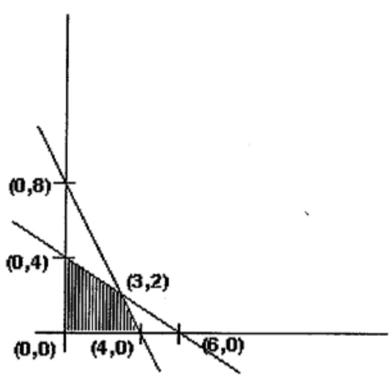
$$2x + 3y \leq 12$$
$$x \geq 0, y \geq 0$$



5. A yard service company has 40 hours of worker time available. Mowing a lawn (x) takes 3 hours and trimming the edges requires 2 hours. The profit from mowing lawns is $15 per hour and the profit from trimming is $10 per hour. Which of the following inequalities represents a resource constraint for this situation?

a. $3x + 2y \leq 40$
b. $\frac{40}{3}x + 10y \leq 40$
c. $15x + 10y \leq 40$
d. $5x + 5y \leq 40$

6. The graph below is a sketch of a feasible region in a linear programming problem. Which point is not in the feasible region?



a. (0,4) b. (4,0) c. (6,0) d. (1,2)

LINEAR PROGRAMMING WORKSHEET 4:
NORTHWEST CORNER, TRANSPORTATION PROBLEMS

ANSWER KEY

1 (1)	7	2	1
3 (1)	4 (2)	5 (1)	4
2	2	1	

Determine the cost associated with the solution you found.

Cost = 1(1) + 1(3) + 2(4) + 1(5) = 1 + 3 + 8 + 5 = 17

1. Compute the indicator value for each non-circled cell.

Cell 1,2 = +7 -1 + 3 – 4 = 6

Cell 1,3 = +2 – 1 + 3 – 5 = -1

2. Apply the Northwest Corner rule to the tableau below:

11 (1)	6 (1)	5	2
1	9 (2)	2 (5)	7
1	3	5	

3. Determine the cost associated with your solution.

Cost = 1(11) + 1(6) + 2(9) + 5(2) = 11 + 6 + 18 + 10 = 45

Compute the indicator value for each non-circled cell.

Cell 1,3 = +5 – 6 + 9 – 2 = +6 Or Cell 1,3 = +5 – 11 + 1 – 2 = -7

Cell 2,1 = +1 – 9 + 6 – 11 = -13 Or Cell 2,1 = +1 - 2 + 5 – 11 = -7 (Same option as above)

Compute the indicator value for each non-circle cell.

Cell 1,3 = +5 – 6 + 9 – 2 = +6 (or) Cell 1,3 = +5 – 11 + 1 – 2 = -7

Cell 2,1 = +1 – 9 + 6 – 11 = -13 (or) Cell 2,1 = +1 – 2 + 5 – 11 = -7 (same option as above)

Linear Programming Worksheet #4

Northwest Corner, Transportation Problems

1. Apply the Northwest Corner rule to the accompanying tableau, which arose from meeting the demands of fruit stands for peaches from supplies available from local orchards.

			Supplies
1	7	2	1
3	4	5	4
2	2	1	
Demands			

Determine the cost associated with the solution you found.

1. Compute the indicator value for each non-circled cell.

2. Apply the Northwest Corner rule to the tableau below:

			Supplies
1	7	2	1
3	4	5	4
2	2	1	
Demands			

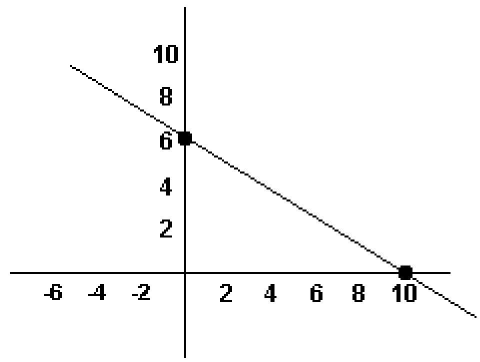
3. Determine the cost associated with your solution.

Compute the indicator value for each non-circled cell.

SAMPLE EXAM QUESTIONS WITH SOLUTIONS

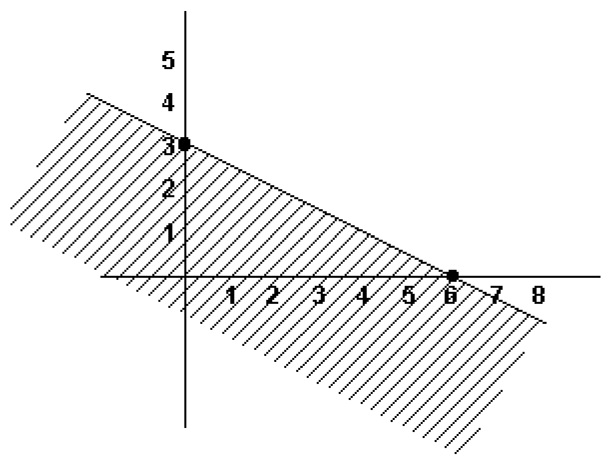
1. Sketch the graph of the equation $3x + 5y = 30$.

   Ans:



2. Sketch the graph of the inequality $2x + 4y \leq 12$.

   Ans:



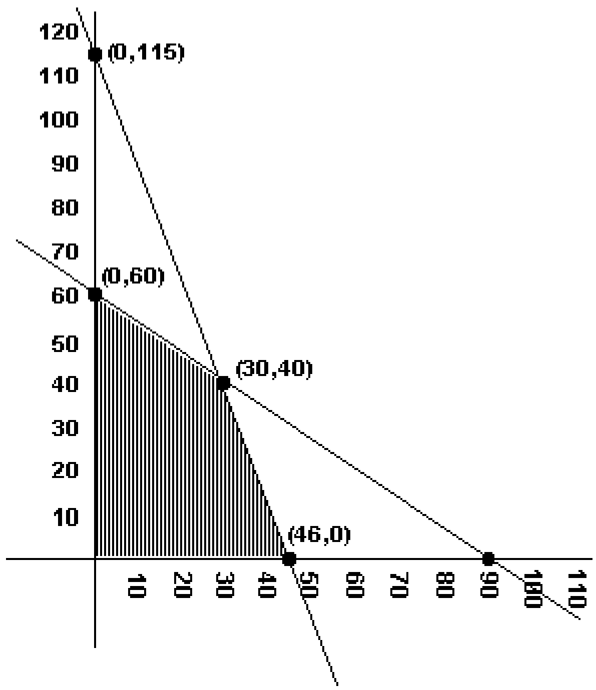
3. Find the point of intersection for the lines represented by the equations $2x + 7y = 61$ and $3x + 4y = 46$.

   Ans: (6,7)

4. With the given the constraints for the following linear programming mixture problem, graph the feasible region.

$2\,x + 3y \leq 180$
$5\,x + 2y \leq 230$
$x \geq 0$
$y \geq 0$

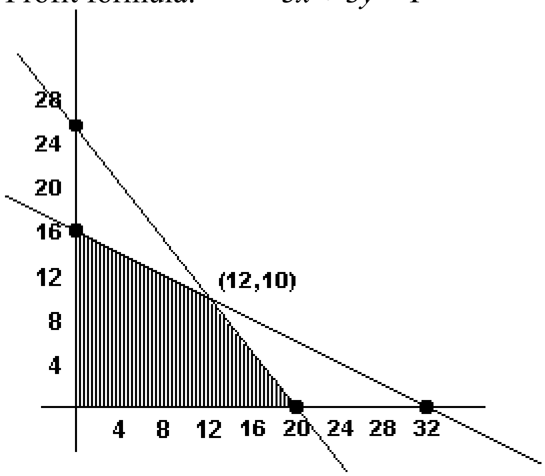
Ans:



5. Solve this linear programming mixture problem: Kim and Lynn produce pottery vases and bowls. A vase requires 25 oz. of clay and 5 oz. of glaze. A bowl requires 20 oz. of clay and 10 oz. of glaze. There are 500 oz. of clay available and 160 oz. of glaze available. The profit on one vase is $5 and the profit on one bowl is $3.

   Ans: x is the number of vases and y is the number of bowls.
   Constraint inequalities:
   $25x + 20y \le 500$
   $5x + 10y \le 160$
   $x \ge 0$
   $y \ge 0$

   Profit formula: $5x + 3y = P$



6. Explain why a linear programming mixture problem might have minimum constraints other than zero.

   Ans: Answers may vary. A linear programming problem might have minimum constraints other than 0 if the company had standing orders for some number of its products that must be filled and cannot be canceled.

7. What type of solution does the Northwest Corner Rule produce for the transportation problem?

   Ans: feasible, but probably not optimal

8. A solution for the transportation problem is optimal when the empty cells have what property?
   Ans: Every empty cell's indicator value is positive or zero

LINEAR PROGRAMMING SAMPLE EXAM

1. Sketch the graph of the equation $3x + 5y = 30$.

2. Sketch the graph of the inequality $2x + 4y$ □ 12.

3. Find the point of intersection for the lines represented by the equations $2x + 7y = 61$ and $3x + 4y = 46$.

4. With the given the constraints for the following linear programming mixture problem, graph the feasible region.

   $2x + 3y$ □ 180
   $5x + 2y$ □ 230
   x □ 0
   y □ 0

5. Solve this linear programming mixture problem: Kim and Lynn produce pottery vases and bowls. A vase requires 25 oz. of clay and 5 oz. of glaze. A bowl requires 20 oz. of clay and 10 oz. of glaze. There are 500 oz. of clay available and 160 oz. of glaze available. The profit on one vase is $5 and the profit on one bowl is $3.

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