Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School[1]

Jo Boaler, Stanford University
Megan Staples, University of Connecticut

To appear in *Teachers' College Record*

Abstract

The low and inequitable mathematics performance of students in urban American high schools has been identified as a critical issue contributing to societal inequities. In an effort to better the field's understanding of equitable and successful teaching, we report results from a five-year longitudinal study of approximately 700 students as they progressed through three high schools. One of the findings of the study was the important success of "Railside" school, where the mathematics department had detracked classes some years ago and taught through a reform-oriented approach. At Railside school students learned more, enjoyed mathematics more and progressed to higher mathematics levels. This paper presents large-scale evidence of these important achievements and provides detailed analyses of the ways that the Railside teachers brought them about, with a focus on the teaching and learning interactions within the classrooms.

Introduction

The low and inequitable mathematics performance of students in urban American high schools has been identified as a critical issue contributing to societal inequities (Moses & Cobb, 2001) and poor economic performance (Madison & Hart, 1990). Thousands of students in the United States and elsewhere struggle through mathematics classes, experiencing repeated failure. Students often disengage from mathematics, finding little intellectual challenge as they are asked only to memorize and execute routine procedures (Boaler, 2002a). Relatively few are offered opportunities to connect different mathematical ideas and apply methods to different situations. The question of how best to teach mathematics remains controversial and debates are dominated by ideology and advocacy (Rosen, 2001). It is critical that researchers gather more evidence on the ways that mathematics may be taught more effectively, in different settings and circumstances. This paper reports upon one study that may contribute to the growing portfolio of evidence that the field is producing.

In this paper we report upon a five-year longitudinal study of approximately 700 students as they progressed through three high schools. The study comprised a range of qualitative and quantitative research methods including assessments, questionnaires and interviews, conducted every year, and over 600 hours of classroom observations. One of the findings of the study was the important success of one of the schools. At "Railside" school students learned more, enjoyed mathematics more and progressed to higher mathematics levels. What made this result more important was the fact that Railside is an urban school on what locals refer to as the 'wrong' side of the tracks. Trains pass just feet away from the students' desks, interrupting lessons at regular intervals. Students come from homes with few financial resources and the population is culturally and linguistically diverse, with many English language learners. At the beginning of high school the Railside students were achieving at significantly lower levels than the students at the other two more suburban schools in our study. Within two years the Railside students were significantly outperforming students at the other schools. The students were also more positive about mathematics, they took more mathematics courses and many more of them planned to pursue mathematics in college. In addition, achievement differences between students of different ethnic groups were reduced in all cases and were eliminated in most. By their senior year 41% of Railside students were taking advanced classes of pre-calculus and calculus compared to approximately 27% of students in the other two schools. At Railside mathematics classes had a high work-rate and few behavioral problems, and the ethnic cliques that form in many schools were not evident. In interviews, the students told us that they learned to respect students from other cultures and circumstances through the approach used in their mathematics classes. The mathematics teachers at Railside achieved something important that many other teachers could learn from – they organized an effective instructional program for students from traditionally marginalized backgrounds and they taught students to enjoy mathematics and to include it as part of their futures. This paper will present evidence of these important achievements and report upon the ways that the teachers brought them about.

Research on Equitable Teaching

Students' opportunities to learn are significantly shaped by the curriculum used in classrooms and by the decisions teachers make as they enact curriculum and organize other aspects of instruction (Darling-Hammond, 1998; Boaler, 2002b). Studies that have monitored the impact of conceptually oriented mathematics materials, taught well and with consistency, have shown higher and more equitable results for participating students than procedure-oriented curricula taught using a demonstration and practice approach (see for example, Boaler, 1997, 2000; Briars & Resnick, 2000; Schoenfeld, 2002; Silver, Smith & Nelson, 1995). Such findings support a widely held belief that *reform* curricula (which we discuss in more detail below) hold the potential for more equitable outcomes (Schoenfeld, 2002; Boaler, 2002b). But studies of the enactment of reform-oriented curricula have also shown that such approaches can be difficult to implement and that such curricula are unlikely to counter inequities unless accompanied by particular teaching practices (Boaler, 2002a, 2002b; Lubienski, 2000). The demands placed upon students in reform-oriented classrooms are quite different from those in more traditionally organized classrooms (Chazan, 2000; Corbett & Wilson, 1995; Lampert, 2001; Lubienski, 2000). There are some indications that the success of reform-oriented approaches depend on teachers' careful and explicit attention to the ways students may be helped to participate in new learning practices (Boaler, 2002a, 2002b; Cohen & Ball, 2001; Corbett & Wilson, 1995) as well as the teachers' social and cultural awareness and sensitivity (K. Gutiérrez, Baquedano-Lopez, & Tejeda, 1999; R. Gutiérrez, 1999).

The need for teachers to explicitly attend to students' understanding of the ways they need to work is consistent with a broad research literature on formative assessment. The main tenets of formative assessment are that students must have a clear sense of the characteristics of high quality work, a clear sense of the place they have reached in their current work, and an understanding of the steps they can take to close the gap between the two (Black & Wiliam, 1998). The idea that careful attention needs to be paid to students' awareness of expected ways of working is also supported by the work of Delpit (1988). Lisa Delpit (1988) has argued that teachers must make explicit the unarticulated rules governing classroom interactions that support different schooling practices, and students must be given opportunities to master those ways of being, doing and knowing. To not support students in *code switching* (Heath, 1983) is to participate in perpetuating inequality.

Many researchers have documented the importance of cultural sensitivity and awareness among teachers. Some researchers have highlighted the value of redesigning curricular materials based on students' cultures or out-of-school practices (Lee, 1995; Tharpe & Gallimore, 1988). In some instances redesign has involved developing curricular examples and schooling structures that build upon the cultural resources students bring to school. Lee (1995), for example, developed an English course which built upon African-American students' competence with social discourse (specifically, the practice of signifying), by focusing on song lyrics. She used this as a bridge into the study of other poetry, discussions of literary interpretation, and as a basis for students' writing. Lee described this approach as "a model of cognitive apprenticing based on cultural foundations" (p. 162). This form of cognitive apprenticeship produced achievement gains in the experimental group that were over twice the gains of the control group. Tharpe and Gallimore (1988) worked with native Hawaiians in their Kamehameha Elementary Education Program (KEEP), designing the structure of the school day and classroom activities to be consonant with the students' home cultures. Their research on this program has consistently demonstrated learning gains for this traditionally marginalized group of children that meet or surpass the average gains of the population as a whole. Ladson-Billings' (1994, 1995) description of *culturally relevant teaching* also highlights the importance of teachers understanding culture and promoting a flexible use of students' local, national and global cultures. Ladson-Billings locates this dimension of teachers' work within a broad description of good teaching which includes features such as subject matter knowledge, pedagogical knowledge, notions of academic achievement, and assessment.

In other instances researchers have found that teaching approaches are more equitable when teachers are sensitive to the cultural differences of their students, without necessarily basing curricular examples upon the students' cultures or aligning instruction with students' out-of-school practices. Rochelle Gutiérrez (1996, 1999, 2000) for example, found that mathematics departments committed to equity enhanced the success of students even when they did not speak the students' languages, nor did they design particular curricular examples to be culturally sensitive. They did, however, use innovative instructional practices and provide a rigorous and common curriculum for all students. Kris Gutiérrez (1995; Gutiérrez, Larson & Kreuter, 1995) documented the use of a *third space* by a teacher who was successful in supporting broad participation across a range of students. In classrooms, often the only valid "space" for participation is within a more formal, structured agenda that is defined by the teacher. A third space can be created when the teacher takes up a student's proposal or idea that, at least on the surface, is not closely connected to the academic concepts or topics at hand. The creation of a third space allows students to influence the agenda and course of lessons, and allows the teacher to build upon students' prior experiences, creating a classroom culture that supports a wider range of participation practices. Hand (2003) found support for the importance of this practice in her study of three high school teachers from Railside school (the focus of this article). These studies collectively imply that teaching practices that evince social awareness and cultural sensitivity are critical if the desired outcome is student participation and academic success.

Research on ability grouping also sheds light on the nature of teaching approaches that are more equitable. A consistent finding across studies on ability grouping is that students in lower groups are offered restricted curricular diets that severely limit their opportunities to learn

(Boaler, 1997; Knapp, Shields & Turnbull, 1992; Oakes, 1985). Lower track classes, disproportionately populated by students of lower socioeconomic status and ethnic minority students, maintain or produce inequities in schools as classes are taught by less well qualified teachers and teachers who often have low expectations for their students (Oakes, 1985). Mixed ability approaches to teaching have consistently demonstrated more equitable outcomes (Boaler, 1997; Cohen & Lotan, 1997; Linchevski & Kutscher, 1998).

We conducted our study of student learning in different schools with the knowledge that a multitude of schooling variables—ranging from district support and departmental organization (Talbert & McLaughlin, 1996) to curricular examples and classroom interactions—could impact the learning of students and the promotion of equity. This helped direct our attention as we conducted a longitudinal, five-year study of the different factors impacting the mathematics learning of 700 high school students from different cultures and social classes who were taught in very different ways. Our study centered upon the affordances of different curricula and the ensuing teaching and learning interactions in classrooms. It also considered the role of broader school factors and the contexts in which the different approaches were enacted.

Description of the Study

The Schools and Students

The Stanford Mathematics Teaching and Learning Study was a five-year, longitudinal study of three high schools with the following pseudonyms: Greendale, Hilltop and Railside. These three schools are reasonably similar in terms of their size, and share the characteristic of employing committed and knowledgeable mathematics teachers. They differ in terms of their location and student demographics. (See Table 1[1].)

Railside High School, the focus of this analysis, is situated in an urban setting. The school is located on the eastern side of a medium sized city of approximately 60,000 residents. Lessons are frequently interrupted by the noise of trains passing just feet away from the classrooms. Railside has a diverse student population with students coming from a variety of ethnic and cultural backgrounds. Hilltop High School is situated in a more rural setting, and approximately half of the students are Latino and half white. Greendale High School is situated in a coastal community with very little ethnic or cultural diversity (almost all students are white).

Table 1
Schools, Students & Mathematics Approaches

	Railside	Hilltop	Greendale
Enrollment (approx.)	1500	2000	1200
Study demographics	40% Latino/a 25% African Am. 15% White 15% Asian/Pac. Islanders 5% other ethnicities	60% White 35% Latino/a 5% other ethnicities	90% White 5% Latino/a 5% other ethnicities

[1] All data given in this paper, that have been obtained from websites, have been rounded to preclude identification of the schools.

ELL[1] students	30%	20%	0%
Free/reduced lunch	30%	20%	10%
Parent education, % college grads	20%	30%	40%
Mathematics curriculum approaches	Teacher designed reform-oriented curriculum, conceptual problems, groupwork	Choice between "traditional" (demonstration and practice, short problems) and IMP (group work, long, applied problems)	Choice between "traditional" (demonstration and practice, short problems) and IMP (group work, long, applied problems)

[1] ELL is English Language Learners

The three high schools were chosen because they enabled us to observe and study three different mathematics teaching approaches. Case selection then was *purposive* (Yin, 1994). Both Greendale and Hilltop schools offered students (and parents) a choice between a traditional sequence of courses, taught using conventional methods of demonstration and practice, and an integrated sequence of courses in which students worked on a more open, applied curriculum called the Interactive Mathematics Program (Fendel, Fraser, Alper, & Resek, 2003), or IMP. Students in IMP classes worked in groups and spent much more time discussing mathematics problems than those in the traditional classes. Railside school used a reform-oriented approach and did not offer a choice. The teachers worked collaboratively and they had designed the curriculum themselves, drawing from different reform curricula such as the College Preparatory Mathematics Curriculum (Sallee, Kysh, Kasimatis, & Hoey, 2000) and IMP. In addition to a common curriculum, the teachers also shared teaching methods and ways of enacting the curriculum. As they emphasized to us, their curriculum could not be reduced to the worksheets and activities they gave students. Mathematics was organized into the traditional sequence of classes — algebra followed by geometry, then advanced algebra and so on — but the students worked in groups on longer, more conceptual problems.

Another important difference between the classes in the three schools we studied was the heterogeneous nature of Railside classes. Whereas incoming students in Greendale and Hilltop could enter geometry or could be placed in a remedial class, such as 'math A' or 'business math', all students at Railside entered the same algebra class. The department was deeply committed to the practice of mixed ability teaching and to giving all students equal opportunities for advancement. The teachers at Railside strived to ensure that good teaching practices were shared; one way in which this was achieved was through something that the department calls "following." The co-chairs structured teaching schedules so that a new teacher could stay a day or two behind a more experienced teacher, allowing the new teacher to observe lessons and activities during her daily preparation period before she tried to adapt it for her classrooms (Horn, 2002, 2005).

We monitored three approaches in the study - 'traditional' and 'IMP' (as labeled by the two schools) and the 'Railside approach.' However, as only one or two classes of students in Greendale and Hilltop chose the IMP curriculum each year, there were insufficient numbers of students to include in our statistical analysis. The main comparison groups of students in the study were therefore approximately 300 students who followed the traditional curriculum and teaching approaches in Greendale and Hilltop schools and approximately 300 students at Railside who were taught using reform-oriented curriculum and teaching methods. These two groups of students[2] provide an interesting contrast as they experienced the same content, taught in very different ways. Class sizes were similar across the schools. During Year 1, there were approximately 20 students in each math class, in line with the class-size reduction policy that was in place in California at that time. In Years 2 and 3, classes were slightly larger at the schools, but generally ranged from 25-35 students.

Research Methods

Given our goal of understanding the highly complex phenomena of teaching and learning mathematics, we gathered a wide array of data, both qualitative and quantitative. Data were collected to inform our understanding of the teaching approaches and classroom interactions, students' views of mathematics, and student achievement. Each data source (lesson observations, interviews, videos, questionnaires, assessments) was analyzed separately using standard procedures of coding and/or statistical analysis. The findings from these multiple sources were then analyzed and understood in relation to one another, thus illuminating trends and themes across sources and affording the opportunity to triangulate the data.

We were greatly aided in our analytic process by having a team of researchers. Each investigator brought an informed perspective that enhanced our discussions at weekly team meetings. With few exceptions, a minimum of two researchers analyzed each portion of data and results reported to the team for review. The themes reported here were agreed upon by the team which increases our confidence in the validity of our analyses and findings (Eisenhart, 2002). We also shared the analyses with the teachers as a form of *member check* (Glesne & Peshkin, 1992), further enhancing the validity of the findings. Communication with the teachers at Railside was extensive, and included yearly presentations to the mathematics department on our findings and interpretations of analyses. In the remainder of this section, we describe each of the different kinds of data collected as part of this multi-faceted study.

Classroom observations and teaching approaches.

To monitor and analyze the teaching practices in the three schools we observed approximately 600 hours of lessons, many of which were videotaped. These lessons were analyzed in three different ways. First, we drew upon our observations from class visits and videotapes to produce *thick descriptions* (Geertz, 2000) of the teaching and learning in the different classes. We also identified one or two focal teachers for each approach in each school, and developed analyses of their teaching, focusing on "teacher moves" that shaped students' engagement with mathematics and mathematical activity. These focal cases were based on classroom observations and analyses of videos of lessons. At Railside, over a three year time period, eight teachers served as focal cases, giving us insights into the similarities and differences in the teachers' practices. The remaining Railside teachers were also observed but did not serve as focal teachers and were not videotaped.

Second, we conducted a quantitative analysis of time allocation during lessons. A mutually exclusive set of categories of the ways in which students spent time in class was developed, which included such categories as teacher talking, teacher questioning whole class, students working alone, and students working in groups. When agreement was reached on the categories, three researchers coded lessons until over 85% agreement was reached. We then completed the coding of over 55 hours of lessons, coding every 30-second period of time. This yielded 6,800 coded segments. We also recorded the amount of time that was spent on each mathematics problem in class. This coding exercise was only performed on Year 1 classes (traditional algebra, Railside algebra, and IMP 1) as it was extremely time intensive and we lacked the resources to perform the same analysis every year.

Third, in addition to these qualitative and quantitative analyses of lessons, we performed a detailed analysis of the questions teachers asked students dividing their questions into such categories as *probing*, *extending* and *orienting*. This level of analysis fell between the qualitative and quantitative methods we had used and was designed in response to our awareness that the teachers' questions were an important indicator of the mathematics on which students and teachers worked (see Boaler & Brodie, 2004). Our coding of teacher questions was more detailed and interpretive than our coding of instructional time but it was sufficiently quantitative to enable comparisons across classes. Our coding of videos and the development of cases for focal teachers provided a strong foundation for understanding differences in the approaches. We also interviewed teachers from each approach at various points in the study although the teachers' perspectives on their teaching were not a major part of our analyses.

Our ongoing analyses, along with our experiences in the schools, also informed our design of interview questions and questionnaires, which further contributed to the development of the themes by which we analyzed the data. Comparisons across cases then led to the identification of important characteristics of the Railside approach, which we report in the findings section.

Students' beliefs and relationships with mathematics.

In order to consider students' experiences of mathematics class and their developing beliefs about mathematics we interviewed at least 60 students in each of the four years that students attended high school. This helped us to consider and analyze the ways the different approaches influenced students' developing relationships with mathematics (see also Boaler, 2002c). Students were typically interviewed in same-sex pairs and we sampled high and low achievers from each approach in every school, taking care to interview students from different cultural and ethnic groups. We also administered questionnaires to all of the students in the focus cohorts in Years 1, 2 and 3 of the study, when most students were required to take mathematics. The questionnaires combined closed, Likert-response questions with more open-ended questions. The questionnaires asked students about their experiences in class, their enjoyment of mathematics, and their perceptions about the nature of mathematics and learning. Two or more researchers coded interviews and open responses to questionnaires. Likert-scale questionnaire items were analyzed using factor analysis. The observations, interviews and questionnaires combined to give us information on the teaching and learning practices in the different approaches and students' responses to them.

Student achievement data.

In addition to monitoring the students' experiences of the mathematics curricula, we assessed their understanding of math content in a range of different ways, including content-aligned tests and open-ended project assessments during Years 1, 2, and 3 of the study, the years when most of the students took mathematics. The content-aligned tests and open-ended project assessments were carefully written by the research team and reviewed by the teachers in each approach (traditional, Railside, and IMP) to make sure they fairly assessed curricula and instruction. Only content common to the three approaches was included and an equal proportion of question-types from each of the three teaching approaches were used on the content-aligned tests. The first assessment we administered was given at the beginning of high school. It was a test of middle school mathematics, which students were expected to know at that time, and it served as a baseline assessment. The second assessment was given at the end of Year 1 and it evaluated only algebraic topics that the students had encountered in common across the different approaches. At the beginning of Year 2 we administered the same assessment, giving us a record of the achievement of all students starting Year 2 classes. The Year 2 assessment evaluated algebra and geometry, as did the Year 3 assessment, although the Year 3 questions included more advanced algebraic material.

The open-ended project assessments we developed were longer, more applied problems that students were given to work on in groups. These problems were administered in Years 1, 2 and 3 and they were given to one class in each approach in each school, and the different groups were videotaped as they worked (see Fiori & Boaler, 2004). We also gathered data on the students' scores on state administered tests. Specifically, data from the CAT6, a standardized state assessment, and the California Standards Test of algebra were collected for each school.

Results

In this section, we first report the findings about the two teaching approaches (traditional and Railside), student achievement and attainment data, and student perceptions of the different approaches and of mathematics. In subsequent sections, we analyze the source of the Railside students' success, as demonstrated on a number of different indicators, by unpacking a number of the practices characterizing the Railside approach.

The Teaching Approaches

Most of the students in Hilltop and Greendale high schools were taught mathematics using a traditional approach, as described by teachers and students at the two schools – they sat individually, the teachers presented new mathematical methods through lectures, and the students worked through short, closed problems. Our coding of lessons showed that approximately 21% of the time in algebra classes was spent with teachers lecturing, usually demonstrating methods. Approximately 15% of the time teachers questioned students in a whole class format. Approximately 48% of the time students were practicing methods in their books, working individually, and students presented work for approximately 0.2% of the time. The average time spent on each mathematics problem was 2.5 minutes, and students completed an average of 24 problems in one hour class period. Our focused analysis of the types of questions teachers asked, which classified questions into seven categories, was conducted with two of the teachers of traditional classes (325 minutes of teaching). This showed that 97% and 99% of the two teachers' questions in traditional algebra classes fell into the procedural category (Boaler & Brodie, 2004).

At Railside school the teachers posed longer, conceptual problems and combined student presentations with teacher questioning. Teachers rarely lectured and students were taught in heterogeneous groups. Our coding of time spent in classrooms showed that teachers lectured to classes for approximately 4% of the time. Approximately 9% of the time teachers questioned students in a whole class format. Approximately 72% of the time students worked in groups while teachers circulated the room showing students methods, helping students and asking them questions of their work, and students presented work for approximately 9% of the time. The average time spent on each mathematics problem was 5.7 minutes, or an average of 16 problems in a 90-minute class period—less than half the number completed in the traditional classes. Our focused analysis of the types of questions teachers asked, conducted with two of the Railside teachers (352 minutes of teaching), showed that Railside math teachers asked many more varied questions than the teachers of traditional classes. Sixty-two percent of their questions were procedural, 17% conceptual, 15% probing, and 6% fell into other questioning categories (Boaler & Brodie, 2004). The broad range of questions they asked was typical of the teachers at Railside who deliberately and carefully discussed their teaching approaches, a practice which included sharing good questions to ask students, as will be described below. We conducted our most detailed observations and analyses in the first-year classes when students were taking algebra, but our observations in later years as students progressed through high school showed that the teaching approaches described above continued in the different mathematics classes the students took.

Student Achievement and Attainment

As noted above, at the beginning of high school we gave all students who were starting algebra classes in the three schools a test of middle school mathematics.[3] At Railside, all incoming students were placed in algebra as the school employed heterogeneous grouping. Comparisons of means indicated that at the beginning of Year 1, the students at Railside were achieving at significantly lower levels than students at the two other schools using the traditional approach ($t = -9.141$, $p < 0.001$, $n = 658$), as can be seen in Table 2. The relatively low performance of the Railside students is not atypical for students in urban, low-income communities (Haberman, 1991). At the end of Year 1 we gave all students a test of algebra to measure what students had learned over the year. Comparisons of means showed that the scores of students in the two approaches were now very similar (traditional = 23.9, Railside = 22.1) a difference that was significant at the 0.04 level ($t = -2.04$, $p = 0.04$, $n = 637$). Thus the Railside students' scores were approaching comparable levels after a year of algebra teaching. At the end of Year 2 we gave students a test of algebra and geometry, reflecting the content the students had been taught over the first two years of school. By the end of Year 2 Railside students were significantly outperforming the students in the traditional approach ($t = -8.304$, $p < 0.001$, $n = 512$).

Table 2
Assessment Results

	Traditional			Railside			
	Mean score	Std Deviation	n	Mean score	Std. Deviation	n	t (level of significance)
Y1 Pre-test	22.23	8.857	311	16.00	8.615	347	-9.141 (*p*<0.001)
Y1 Post-test	23.90	10.327	293	22.06	12.474	344	-2.040 (*p*=0.04)
Y2 Post-test	18.34	10.610	313	26.47	11.085	199	-8.309 (*p*<0.001)

Y3 Post-test	19.55	8.863	290	21.439	10.813	130	-1.75 (*p*=0.082)

There were fewer students in the geometry classes in Railside due to the flexibility of Railside's timetable, which allowed students to choose when they took geometry classes (as will be described in the next section). The students in geometry classes at Railside did not represent a selective group; they were of the same range as the students entering Year 1. Analyses of students who took all three tests, shown in Table 3, which includes only those students who went straight from algebra to geometry in each school (a smaller number), also show that the Railside students started at significantly lower levels and ended Year 2 at significantly higher levels. Interestingly, the students most advantaged by the teaching approach at Railside, compared to those in traditional, tracked classes, appeared to be those who started at the highest levels. These students showed the greatest achievement advantage in Year 2, when compared with students in tracked classes in the other schools, a finding that should alleviate concerns that high attaining students are held back by working in heterogeneous groups. Interview data, reported in the next sections, suggest that the high attaining students developed deeper understanding from the act of explaining work to others.

Table 3.
Scores of students who took Y1 pre-test, Y1 post-test and Y2 post test.

		Y1 Pre-test		Y1 Post-test		Y2 Post-test	
	n[a]	Mean	SD	Mean	SD	Mean	SD
Railside	90	20.58	8.948	29.19	11.804	24.96	10.681
Traditional	163	23.44	8.802	25.86	10.087	16.58	8.712
t (level of significance)		2.463 (p = 0.014)		2.364 (p=0.019)		6.364 (p=0.000)	

[a]n is number of students in all analyses

In Year 3 the students at Railside continued to outperform the other students, although the differences were not significant (t = -1.75, p = 0.082, n = 420). The Railside students' achievement in Year 3 classes may not have been as high in relation to the traditional classes as the Year 3 Railside curriculum had not been developed as much by the department, and the classes were taught by teachers in their first two years of teaching. In Year 4 we did not administer achievement tests as mathematics was taken by a much more selective group at that point in time in all three schools. However, more students at Railside continued to take higher-level math courses than students at Greendale and Hilltop schools. By their senior year, 41% of Railside students were taking advanced classes of pre-calculus and calculus compared to about 27% of students in the other two schools[4].

The Railside mathematics teachers were also extremely successful at reducing the achievement gap between groups of students belonging to different ethnic groups at the school. Table 4 shows significant differences between groups at the beginning of the ninth-grade year, with Asian, Filipino, and White students each outperforming Latino and Black students (p<.001).

Table 4

Railside Year 1 Pre-test Results by Ethnicity

Ethnicity	n	Mean	Median	Std. Dev
Asian	27	22.41	22	8.509
Black	68	12.28	12	6.286
Hispanic/Latino	103	14.28	12	7.309
Filipino	23	21.61	22	8.289
White	51	21.20	21	9.362

At the end of Year 1, only one year after the students started at Railside, there were no longer significant differences between the achievement of white and Latino students, nor Filipino students and Latino and Black students. The significant differences that remained at that time were between white and Black students and between Asian students and Black and Latino students (ANOVA F=5.208; df=280; p=0.000). Table 5 shows these results.

Table 5
Railside Year 1 Post-test Results by Ethnicity

Ethnicity	n	Mean	Median	Std. Dev
Asian	27	29.44	30	12.148
Black	68	18.21	16.50	10.925
Hispanic/Latino	103	21.31	21	11.64
Filipino	23	26.65	26	10.504
White	51	26.69	28	13.626

In subsequent years the only consistent difference that remained was the high performance of Asian students who continued to significantly outperform Black and Latino students, but differences between White, Black and Latino students disappeared. Achievement differences between students of different ethnicities at the other schools remained. In addition, it is worth noting that there were no gender differences in performance in any of the tests we gave students at any level, and young women were well represented in higher mathematics classes. They made up 50% of students in the advanced classes at Hilltop, 48% at Greendale and 59% at Railside.

Student Perceptions and Relationships with Mathematics

In addition to high achievement, the students at Railside also enjoyed mathematics more than the students in the other approach. In questionnaires given to the students each year, the Railside students were always significantly more positive about their experiences with mathematics. For example, 71% of Railside students in Year 2 classes (n =198), reported ‘enjoying math class’ compared with 46% of students in traditional classes (n=318) (t = -4.934; df=444.62; p<0.001). In the Year 3 questionnaire students were asked to finish the statement: ‘I enjoy math in school’ with one of four time options: all of the time, most of the time, some of the time, or none of the time. Fifty-four percent of students from Railside (n=198) said that they enjoyed mathematics all or most of the time, compared with 29% of students in traditional classes (n=318) which is a significant difference (t = 4.758; df = 286; p<0.001). In addition, significantly more Railside students agreed or strongly agreed with the statement ‘ I like math’, with 74% of Railside students responding positively, compared with 54% of students in traditional classes ($t = -4.414$; $df=220.77$; $p<0.001$).

In Year 4 we conducted interviews with 105 students in the three different approaches. Most of the students were seniors and they were chosen to represent the breadth of attainment displayed by the whole school cohort. These interviews were coded and students were given scores on the categories of *interest*, *authority*, *agency* and *future plans for mathematics.* The first three are themes that emerged from our data, which we targeted in our final interviews with students. In addition, we were interested in the students' future plans with mathematics as students were completing high school. The categories of *authority* and *agency* (Holland, Lachiotte, Skinner, & Cain, 1998) emerged as important as students in the different approaches varied in the extent to which they believed they had authority (the capacity to validate mathematical methods and ideas using their own knowledge rather than the teacher or textbook) or that they could work with agency (having the opportunity to inquire and use their own ideas) (see Boaler & Gresalfi, in preparation). Significant differences were found in all of these categories with the students at Railside being significantly more interested in mathematics ($\lambda^2 = 12.806$, $df = 2$, $p = 0.002$, $n= 67$) and believing they had significantly more authority ($\lambda^2 = 29.035$, $df = 2$, $p = 0.000$, $n= 67$) and agency ($\lambda^2 = 22.650$, $df = 2$, $p = 0.000$, $n= 63$). In terms of future plans, *all* of the students interviewed at Railside intended to pursue more mathematics courses compared with 67% of students from the traditional classes, and 39% of Railside students planned a future in mathematics compared with 5% of students from traditional classes ($\lambda^2 = 18.234$, $df = 2$, $p = 0.000$, $n= 65$).

Because of the challenges of accessing individual student data, as opposed to whole-school data, due to student confidentiality issues, we are unable to report anything beyond school scores for the students on state administered tests. Despite this limitation, these school-level data are interesting to examine and raise some important issues with respect to testing and equity, as Railside students performed higher on our tests, district tests and the California Standards test of algebra but did not fair as well on the CAT 6, a standardized test, nor on indicators of adequate yearly progress (AYP) which are determined primarily by standardized tests.

The California Standards test, a curriculum-aligned test taken by students who had completed algebra, showed the Railside students scoring at higher levels than the other two schools (see Table 6). Forty-nine percent of Railside students scored at or above the basic level, compared to 33% at Greendale and 41% at Hilltop.

Table 6
California Standards Test, Algebra, 2003. Percent of students attaining given levels of proficiency.

	Greendale	Hilltop	Railside
n	125	224	188
Advanced	0	0	0
Proficient	5	15	15
Basic	30	30	35
Below basic	55	45	35
Far below basic	10	15	15

In contrast, students at Hilltop and Greendale scored at higher levels on the CAT 6, and these schools had higher AYP numbers, as seen in Tables 7 and 8.

Table 7
CAT 6, 2003, STAR, Grade 11 (Year 3): Percent of students at or above 50th percentile

	Railside	Hilltop	Greendale
n	341	436	257
Reading	40	55	75
Language	30	55	70
Mathematics	40	50	70

Table 8
AYP (adequate yearly progress), 2003: Percent of students 'proficient' at language arts and mathematics

	Difference (English – Math)	"Similar schools" average difference
Railside	1.3	12.8
Hilltop	9.2	10.5
Greendale	14.5	11.9

The relatively low performance of the Railside students on the state's standardized tests is interesting and may be caused by the cultural and linguistic barriers provided by the state tests. The correlation between students' scores on the language arts and mathematics sections of the AYP tests, across the whole state of California, was a staggering 0.932 for 2004. This data point provides strong indication that the mathematics tests were testing language as much as mathematics. This argument could not be made in reverse as the language tests do not contain mathematics. Indeed the students at Railside reported in open-ended interviews that the standardized tests used unfamiliar terms and culturally biased contexts that our tests did not use (see also Boaler, 2003). Tables 7 and 8 also show interesting relations between mathematics and language as the Greendale and Hilltop school students were more successful on tests of reading and language arts, a trend that held across the state, but the Railside students were as or more successful on mathematics. Another interesting result to note is that 37% more White students scored at or above the 50th percentile than Latino students at Hilltop (the only other sizeable group of ethnic minority students in the study) on the CAT 6. At Railside the difference between the same two groups was only 9%. The data in Tables 6- 8 may indicate the inability of the state standardized tests to capture the mathematical understanding of the Railside students that was demonstrated in many other formats. The test data do need to be approached with caution, however, as the cohort we studied did not need to pass the high school exit examination in order to graduate, and the STAR tests that are used in California were taken by only a small proportion of students.

Summary Comments

The students at Railside school enjoyed mathematics more than students taught more traditionally, they achieved at higher levels on curriculum-aligned tests, and the achievement gap between students of different ethnic and cultural groups was lower than those at the other schools. In addition, the teachers and students achieved something that Boaler (2006) has termed *relational equity*. In studying equity most researchers look for reductions in achievement differences for students of different ethnic and cultural groups and genders when tests are taken. But Boaler has argued that a goal for equity should also be the creation of classrooms in which students learn to treat each other equitably, showing respect for students of different cultures, genders and social classes. Schools are places where students learn ways of acting and being that they are likely to replicate in society, making respect for students from different circumstances an important goal. It is not commonly thought that mathematics classrooms are places where students should learn about cultural respect but students at Railside reported that they learned to value students who came from very different backgrounds to themselves because of the approach of their mathematics classes, as we will describe shortly (for more detail, see Boaler, 2006).

Analyzing the Sources of Success

Part I. The Department, Curriculum and Timetable

Railside school has an unusual mathematics department. During the years of our study, twelve of the thirteen teachers worked collaboratively, spending vast amounts of time designing curricula, discussing teaching decisions and actions, and generally improving their practice through the sharing of ideas. A study conducted by Horn (2002) on the ways in which the department collaborated found that the teachers spent around 650 minutes a week planning, individually and collectively (their paid work week provides 450 minutes of preparation time, only a small portion of which is used for planning, as teachers must also grade, manage paper work, give extra help and attend to other school obligations). Unusually for the United States, the mathematics department strongly influenced the recruitment and hiring of teachers, enabling the department to maintain a core of teachers with common philosophies and goals. The teachers shared a strong commitment to the advancement of equity and the department had spent many years working out a coherent curriculum and teaching approach that teachers believed enhanced the success of all students. The mathematics department had focused their efforts in particular upon the introductory algebra curriculum that all students take when they start the school. The algebra course is designed around key concepts with questions from various published curricula such as CPM, IMP and a textbook of activities that use algebra LabGear™ (Picciotto, 1995). A theme of the algebra and subsequent courses is multiple representations, and students were frequently asked to represent their ideas in different ways, using *math tools* such as words, graphs, tables and symbols. In addition, connections between algebra and geometry were emphasized even though the two areas were taught in separate courses.

Railside followed a practice of 'block scheduling' and lessons were 90 minutes long, with courses taking place over half a school year, rather than a full academic year[5]. In addition, the introductory algebra curriculum, generally taught in one course in US high schools, including Greendale and Hilltop, was taught in the equivalent of two courses at Railside. The teachers spread the introductory content over a longer period of time partly to ensure that the foundational mathematical ideas were taught carefully with depth and partly to ensure that particular norms – both social and socio-mathematical (Yackel & Cobb, 1996) – were carefully established. The fact that mathematics courses were only half a year long at Railside may appear unimportant, but in fact this organizational decision had a profound impact upon the students' opportunities to take higher-level mathematics courses. In most US high schools, including Greendale and Hilltop, mathematics classes are one year long and a typical student begins with algebra. This means that students cannot take calculus unless they are advanced, as the standard sequence of courses is algebra, geometry, advanced algebra then pre-calculus. Furthermore, if a student fails a course at any time she is knocked out of that sequence and has to retake the course, further limiting the level of content she will reach. In contrast, at Railside the students could take two mathematics classes each year. This meant that students could fail classes, start at lower levels, and/or choose not to take mathematics in a particular semester and still reach calculus. This relatively simple scheduling decision was part of the reason that significantly more students at Railside took advanced level classes in school than students in the other two schools.

As the teachers at Railside were deeply committed to equity and to heterogeneous teaching, they had worked together over the past decade to develop and implement a curriculum that afforded multiple points of access to the mathematics and comprised a variety of cognitively demanding tasks. The curriculum was organized around units that had a unifying theme such as "What is a linear function?" This differs markedly from more standard textbooks where the units are organized around algebraic and other mathematical techniques (e.g., graphing linear functions; factoring polynomials). This organization of the Railside curriculum provided thematic coherence across a set of activities, which afforded students the opportunity to make connections and gave teachers the opportunity to highlight and teach for those connections.

As they developed the curriculum, the department placed a strong emphasis on creating problems that satisfy the criterion of *groupworthy*. Groupworthy problems are those that "illustrate important mathematical concepts, allow for multiple representations, include tasks that draw effectively on the collective resources of a group, and have several possible solution paths" (Horn, 2005, p. 22). Appendix A includes an example of a problem that the department deemed groupworthy.

An important feature of the Railside approach we studied, that cannot be seen in the curriculum materials, was the act of asking follow up questions. For example, when students found the perimeter of a figure (see appendix A) with side lengths represented algebraically, as 10x + 10, the teacher asked a student in each group, "Where's the 10?" requiring that students relate the algebraic equation to the figure. Although the tasks provided a set of constraints and affordances (Greeno & MMAP, 1997), it was in the implementation of the tasks that the learning opportunities were realized (Stein, Smith, Henningsen & Silver, 2000). Teachers' questions significantly shaped the course of implementation. The question of "Where's the 10?" for example was not written on the students' worksheets, but was part of the curriculum, as teachers agreed upon the follow up questions they would ask of students.

Research studies in recent years have pointed to the importance of school and district contexts in the support of teaching reforms (McLaughlin & Talbert, 2001; Siskin, 1994; Talbert & McLaughlin, 1996). Such support is undoubtedly important but Railside is not a case of a district or school that initiated or mandated reforms. The reforms put in place by the mathematics department were supported by the school and were in line with other school reforms but they were driven by the passion and commitment of the mathematics teachers in the department. The school, in many ways, provided a demanding context for the reforms, not least because they had been managed by five different principals in six years, and they had been labeled an 'under-performing school' by the state because of low state test scores. The department, under the leadership of two strong and politically astute co-chairs, fought to maintain their practices at various times and worked hard to garner the support of the district and school. While the teachers felt well supported at the end of our study, Railside does not represent a case of a reforming district encouraging a department to engage in new practices. Rather, Railside is a case of an unusual, committed and hard working department that continues to grow in strength through its teacher collaborations and work.

Part II. Groupwork and 'Complex Instruction'

Many mathematics departments in the US employ group work but few are able to report the success of the Railside students or such high rates of work, as groups do not always function well, with some students doing more of the work than others, and some students being excluded or choosing to opt out. At Railside the teachers employed additional strategies to make group work successful. They adopted an approach called *complex instruction* designed by Elizabeth Cohen and Rachel Lotan (Cohen, 1994; Cohen & Lotan, 1997) for use in all subject areas. The system is designed to counter social and academic status differences in classrooms, starting from the premise that status differences do not emerge because of particular students but because of group *interactions*. The approach includes a number of recommended practices that the school employed that we highlight below.

Multidimensional classrooms.

In many mathematics classrooms there is one practice that is valued above all others – that of executing procedures (correctly and quickly). The narrowness by which success is judged means that some students rise to the top of classes, gaining good grades and teacher praise, whilst others sink to the bottom. In addition, most students know where they are in the hierarchy created. Such classrooms are unidimensional – the dimensions along which success is presented are singular. In contrast, a central tenet of the complex instruction approach is what the authors refer to as *multiple ability treatment*. This treatment is based upon the idea that expectations of success and failure can be modified by the provision of a more open set of task requirements that value many different *abilities*. Teachers should explain to students that "no one student will be 'good on all these abilities' and that each student will be 'good on at least one'" (Cohen & Lotan, 1997, p. 78). Cohen and Lotan provide theoretical backing for their multiple ability treatment using the notion of multidimensionality (Rosenholtz & Wilson, 1980; Simpson, 1981).

At Railside the teachers created multidimensional classes by valuing many dimensions of mathematical work. This was achieved, in part, by implementing open problems that students could solve in different ways. The teachers valued different methods and solution paths and this enabled more students to contribute ideas and feel valued. But multiple solution paths were not the only contributions that were valued by teachers. When we interviewed the students and asked them 'what does it take to be successful in mathematics class?' they offered many different practices such as: asking good questions, rephrasing problems, explaining well, being logical, justifying work, considering answers, and using manipulatives. When we asked students in the traditional classes what they needed to do in order to be successful they talked in much more narrow ways, usually saying that they needed to concentrate, and pay careful attention. Railside students regarded mathematical success much more broadly than students in the traditional classes, and instead of viewing mathematics as a set of methods that they needed to observe and remember, they regarded mathematics as a way of working with many different dimensions. The different dimensions that students believed to be an important part of mathematical work were valued in the teachers' interactions with students and the grading system. Not surprisingly, multidimensionality has implications for curriculum, as the nature of the tasks implemented must be such that they support multiple approaches and a varied set of learning practices. Indeed, the teachers at Railside spent a great deal of time developing groupworthy problems, discussed in Part I (Horn, 2002) which supported their work as they strove to support multidimensional classrooms.

The multidimensional nature of the classes at Railside was an extremely important part of the increased success of students. Put simply, *when there are many ways to be successful, many more students are successful*. Railside students were aware of the different practices that were valued and they felt successful because they were able to excel at some of them. Given the current high-stakes testing climate, teachers may shy away from promoting the development of practices outside of procedure execution because they are not needed on state tests, but the fact that teachers at Railside valued a range of practices and more students could be successful in class appears to have made students feel more confident and positive about mathematics. This may have enhanced their success on tests (even when tests assessed a more narrow range of mathematical work) and their persistence with high-level mathematics classes.

The following comments given by students in interviews provide a clear indication of the multidimensionality of classes:

> *Back in middle school the only thing you worked on was your math skills. But here you work socially and you also try to learn to help people and get help. Like you improve on your social skills, math skills and logic skills. (Janet, Y1)*

> *J: With math you have to interact with everybody and talk to them and answer their questions. You can't be just like "oh here's the book, look at the numbers and figure it out."*
> *Int: Why is that different for math?*
> *J: It's not just one way to do it (…) It's more interpretive. It's not just one answer. There's more than one way to get it. And then it's like: "why does it work"? (Jasmine, Y1)*

It is not common for students to report that mathematics is more 'interpretive' than other subjects. The students at Railside recognized that helping, interpreting and justifying were critically valued practices in mathematics classes.

One of the practices that we found to be particularly important in the promotion of equity was justification. At Railside students were required to justify their answers at almost all times. There are many good reasons for this – justification is an intrinsically mathematical practice (RAND, 2002; Martino & Maher, 1999), but this practice also serves an interesting and particular role in the promotion of equity. Many teachers struggle to deal with the wide range of students who attend classes, particularly in introductory classes such as high school algebra, which include students who are motivated with a wealth of prior knowledge as well as those who are less motivated and /or lack basic mathematical knowledge. At Railside school, algebra classes had a remarkably wide achievement gap, but the teachers embraced the diversity they encountered. One practice that helped them support the learning of all students was justification. The practice of justification made space for mathematical discussions that might not otherwise be afforded. Particularly given the broad range of students' prior knowledge, receiving a justification that satisfied an individual was important as explanations were adapted to the needs of individuals, and mathematics that might not otherwise be addressed was brought to the surface.

The following two students give an indication of the role justification played in helping different students:

> *Int: What happens when someone says an answer?*
> *A: We'll ask how they got it.*
> *L: Yeah because we do that a lot in class. (…) Some of the students – it'll be the students that don't do their work, that'd be the ones, they'll be the ones to ask step by step. But a lot of people would probably ask how to approach it. And then if they did something else they would show how they did it. And then you just have a little session! (Ana & Latisha, Y3)*

It is noteworthy that these two students did not describe students as slow, dumb or stupid, as other students in our study did; they talked only about students 'that don't do their work'.

The following boy was achieving at lower levels than other students and it is interesting to hear him talk about the ways he was supported by the practices of explanation and justification:

> *Most of them, they just like know what to do and everything. First you're like "why you put this?" and then like if I do my work and compare it to theirs. Theirs is like super different 'cos they know, like what to do. I will be like – let me copy, I will be like "why you did this?" And then I'd be like: "I don't get it why you got that." And then like, sometimes the answer's just like, they be like "yeah, he's right and you're wrong" But like – why? (Juan, Y2)*

Juan also differentiated between high and low achievers without referring to such adjectives as 'smart' or 'fast', instead saying that some students 'know what to do'. He also made it very clear that he was helped by the practice of justification and that he feels comfortable pushing other students to go beyond answers and explain 'why' their answers are given. At Railside the teachers carefully prioritized the message that each student had two important responsibilities – both to help someone who asked for help, but also to ask if they needed help. Both are important in the pursuit of equity, and justification emerged as an important practice in the learning of a wide range of students.

Roles.

A large part of the success of the teaching at Railside came from the complex, interconnected system in each classroom in which students were taught to take responsibility for each other and were encouraged to contribute equally to tasks. When students were placed into groups they were given a particular role to play, such as *facilitator*, *team captain*, *recorder/reporter* or *resource manager* (Cohen & Lotan, 1997). The premise behind this approach is that all students have important work to do in groups, without which the group cannot function. At Railside the teachers emphasized the different roles at frequent intervals, stopping, for example, at the start of class to remind facilitators to help people check answers or show their work. Students changed roles at the end of each unit of work. The teachers reinforced the status of the different roles and the important part they played in the mathematical work that was undertaken. These roles, and students' engagement with mathematics that was supported by them, contributed to a classroom environment in which everyone had something important to do and all students learned to rely upon each other.

Assigning competence.

An interesting and subtle approach that is recommended within the complex instruction literature is that of *assigning competence*. This is a practice that involves teachers raising the status of students that may be of a lower status in a group, by, for example, praising something they have said or done that has intellectual value, and bringing it to the group's attention; asking a student to present an idea; or publicly praising a student's work in a whole class setting. For example, during a classroom observation at Railside a quiet Eastern European boy muttered something in a group that was dominated by two outgoing Latina girls. The teacher who was visiting the table immediately picked up on what Ivan said, noting, "Good Ivan, that is important." Later when the girls offered a response to one of the teacher's questions the teacher said, 'Oh that is like Ivan's idea, you're building on that'. The teacher raised the status of Ivan's contribution, which would almost certainly have been lost without such an intervention. Ivan visibly straightened up and leaned forward as the teacher reminded the girls of his idea. Cohen (1994) recommends that if student feedback is to address status issues, it must be public, intellectual, specific, and relevant to the group task (p. 132). The public dimension is important as other students learn about the broad dimensions that are valued; the intellectual dimension ensures that the feedback is an aspect of mathematical work; and the specific dimension means that students know exactly what the teacher is praising. This practice is linked to the multidimensionality of the classroom which values a broad range of work and forms of participation. The practice of assigning competence demonstrated the teachers' commitment to equity and to the principle of showing what different students could do in a multifaceted mathematical context.

Teaching students to be responsible for each other's learning.

A major part of the equitable results attained at Railside came from the serious way in which teachers taught students to be responsible for each other's learning. Many schools employ groupwork which, by its nature, brings an element of responsibility, but Railside teachers went beyond this to encourage the students to take the responsibility very seriously. In previous research on approaches that employ groupwork, students generally report that they prefer to work in groups and they list different benefits, but the advantages usually relate to their own learning (see Boaler, 2000, 2002a, 2002b). At Railside students talked about the value groupwork added to their own learning, but their descriptions were distinctly reciprocal as they also voiced a clear concern for the learning of their classmates. For example:

> *Int: Do you prefer to work alone or in groups?*
> *A: I think it'd be in groups, 'cause I want, like people that doesn't know how to understand it, I want to help them. And I want to, I want them to be good at it. And I want them to understand how to do the math that we do. (Amado, Y1)*

Students talked about their enjoyment of helping others and the value in helping each other:

> *It's good working in groups because everybody else in the group can learn with you, so if someone doesn't understand – like if I don't understand but the other person does understand they can explain it to me, or vice versa, and I think it's cool. (Latisha, Y3*

One unfortunate but common side effect of some classroom approaches is that students develop beliefs about the inferiority or superiority of different students. In our other classes students talked about other students as smart and dumb, quick and slow. At Railside the students did not talk in these ways. This did not mean that they thought all students were the same, they did not, but they came to appreciate the diversity of classes and the different attributes that different students offered:

> *Everybody in there is at a different level. But what makes the class good is that everybody's at different levels so everybody's constantly teaching each other and helping each other out. (Zane, Y2)*

The students at Railside not only learned to value the contributions of others, they also developed a responsibility to help each other.

One way in which teachers nurtured a feeling of responsibility was through the assessment system. Teachers graded the work of a group by, for example, rating the quality of the conversations groups had. The teachers also gave both individual and group tests, which took several formats. In one version students worked through a test together, but the teachers graded only one of the individual papers and that grade stood as the grade for all the students in the group. A third way in which responsibility was encouraged was through a practice of asking one student in a group to answer a follow up question after a group had worked on something. If the student could not answer the question the teacher would leave the group to have more discussion and return to ask the same student again. In the intervening time it was the group's responsibility to help the student learn the mathematics they needed to answer the question. This move of asking one member of a group to give an answer and an explanation, without help from their groupmates, was a subtle practice that had major implications for the classroom environment. In the following interview extract the students talk about this particular practice and the implications it holds:

> *Int: Is learning math an individual or a social thing?*
> *G: It's like both, because if you get it, then you have to explain it to everyone else. And then sometimes you just might have a group problem and we all have to get it. So I guess both.*
> *B: I think both - because individually you have to know the stuff yourself so that you can help others in your group work and stuff like that. You have to know it so you can explain it to them. Because you never know which one of the four people she's going to pick. And it depends on that one person that she picks to get the right answer. (Gisella & Bianca, Y2)*

The students in the extract above make the explicit link between teachers asking any group member to answer a question, and being responsible for their group members. They also communicated an interesting social orientation that became instantiated through the mathematics approach, saying that the purpose in knowing individually was not to be better than others but so "you can help others in your group". There was an important interplay between individual and group accountability in the Railside classrooms.

The four practices described– of multidimensionality, group roles, assigning competence and encouraging responsibility are all part of the complex instruction approach. We now review three other practices in which the teachers engaged that are also critical to the promotion of equity. These relate to the challenge and expectations provided by the teachers.

Part III. Challenge and Expectations

High cognitive demand.

The Railside teachers held high expectations for students and presented all students with a common, rigorous curriculum to support their learning. The cognitive demand that was expected of all students was higher than other schools partly because the classes were heterogeneous and no students were precluded from meeting high-level content. Even when students arrived at school with weak content knowledge well below their grade level, they were placed into algebra classes and supported in learning the material and moving on to higher content. Teachers also enacted a high level of challenge in their interactions with groups and through their questioning, for instance, in the earlier example where students found the perimeter of a set of algebra lab tiles to be 10x+10 and the teachers asked students to explain where the +10 came from. Importantly the support that teachers gave to students did not serve to reduce the cognitive demand of the work, even when students were showing signs of frustration. The reduction of cognitive demand is a common occurrence in mathematics classes when teachers help students (Stein et al., 2000). At Railside the teachers were highly effective in interacting with students in ways that supported their continued thinking and engagement with the core mathematics of the problems.

The students at Railside became aware that the teachers demanded high levels of mathematical work and they came to appreciate that demand. When we interviewed students and asked them what it took to be a good teacher, many of them mentioned the high demand placed upon them, for example:

> *She has a different way of doing things. I don't know, like she won't even really tell you how to do it. She'll be like, 'think of it this way'. There's a lot of times when she's just like – 'well think about it' – and then she'll walk off and that kills me. That really kills me. But it's cool. I mean it's like, it's alright, you know. I'll solve it myself. I'll get some help from somebody else. It's cool. (Ana, Y3)*

The following students, in talking about the support teachers provided, also referred to their teachers' push for understanding:

> *Int: What makes a good teacher?*
> *J: Patience. Because sometimes teachers they just zoom right through things. And other times they take the time to actually make sure you understand it, and make sure that you actually pay attention. Because there's some teachers out there who say: 'you understand this?' and you'll be like "yes," but you really don't mean yes you mean no. And they'll be like "OK." And they move on. And there's some teachers that be like – they know that you don't understand it. And they know that you're just saying yes so that you can move on. And so they actually take the time out to go over it again and make sure that you actually got it, that you actually understand this time. (John, Y2)*

The students' appreciation of the teachers' demand was also demonstrated in our questionnaires. One of the questions started with the stem: 'When I get stuck on a math problem, it is most helpful when my teacher …'. This was followed by answers such as 'tells me the answer,' 'leads me through the problem step by step,' and 'helps me without giving away the answer'. Students could respond to each on a four-point scale (SA, A, D, SD). Almost half of the Railside students (47%) *strongly* agreed with the response: "Helps me *without* giving away the answer," compared with 27% of students in the traditional classes at the other two schools (n= 450, t = -4.257; df = 221.418; p<0.001).

Effort over ability.

In addition to challenging through difficult questions that maintained a high cognitive demand, the teachers also gave frequent and strong messages to students about the nature of high achievement in mathematics, continually emphasizing that it was a product of hard work and not of innate ability. The teachers kept reassuring students that they could achieve anything if they put in the effort. This message was heard by students and they communicated it to us in interviews, with absolute sincerity, as the following quote indicates:

> *To be successful in math you really have to just like, put your mind to it and keep on trying – because math is all about trying. It's kind of a hard subject because it involves many things. (…) but as long as you keep on trying and don't give up then you know that you can do it. (Sara, Y1)*

In the Year 3 questionnaires we offered the statement 'Anyone can be really good at math if they try.' At Railside, 84% of the students agreed with this, compared with 52% of students in the traditional classes (n= 473, t = -8.272; df = 451; p<0.001). But the Railside students did not only come to believe that they could be successful. They developed an important practice that supported them in that – the act of persistence. It could be argued that persistence is one of the most important practices to learn in school – one that is strongly tied to success in school as well as in work and life. We have many indications in our data that the Railside students developed considerably more persistence than the other students. For example, as part of our assessment of students we gave them long, difficult problems to work on for 90 minutes in class, which we videotaped. The Railside students were more successful on these problems, partly because they would not give up on them and they continued to try to find methods and approaches even when they had exhausted many. When we asked in questionnaires: 'How long (in minutes) will you typically work on one math problem before giving up and deciding you can't do it?' the Railside students gave responses that averaged 19.4 minutes, compared with the 9.9 minutes averaged by students in traditional classes (n=438, t = -5.641; df = 142.110; p< 0.001). This response is not unexpected given that the Railside students worked on longer problems in classes, but it also gives some indication of the persistence students were learning through the longer problems they experienced.

In the following interview extract the students link this persistence to the question-asking and justification highlighted earlier:

> *A: Because I know if someone does something and I don't get it I'll ask questions. I'm not just going to keep going and not know how to do something.*
> *L: And then if somebody challenges what I do then I'll ask back and I'll try to solve it. And then I'll ask them: "Well how d'you do it?" (Ana & Latisha, Y3)*

Clear expectations and learning practices.

The final aspect of the teachers' practice we highlight relates to the expectations they offered the students. In addition to stressing the importance of effort, the teachers were very clear about the particular ways of working in which students needed to engage. Cohen & Ball (2001) describe ways of working that are needed for learning as *learning practices*. For example, the teachers would stop the students as they were working and talking and point out valuable ways in which they were working. In one observation we witnessed one of the Railside teachers, Guillermo, helping a boy named Arturo. Arturo said he was confused, so Guillermo told him to ask a specific question. As Arturo framed a question he realized what he needed to do and continued with his thinking. Arturo decided the answer was '550 pennies' but then stopped himself, saying 'no, wait, that's not very much'. At that point Guillermo interrupted him:

> Wait, hold on a second, two things just happened there. Number one is, when I said, "What is the exact question?" you stopped to ask yourself the exact question and then suddenly you had ideas. That happens to a lot of students, if they're confused, the thing you have to do is say, "OK what am I trying to figure out?" Like exactly, and like say it. So say it out loud or say it in your head but say it as a sentence. That's number one and number two, then you checked out the answer and you realized the answer wasn't reasonable and that is *excellent* because a lot of people would have just left it there and not said "What, 500 pennies? That's not very much." (Guillermo, math department co-chair)

The teachers also spent time before projects began setting out the valued ways of working, encouraging students to, for example, pick 'tricky' examples when writing a book, one of the projects they completed, and to "show off" the mathematics that they knew. The teachers communicated very clearly to students which learning practices would help them achieve (see also Boaler, 1997, 2002b).

Conclusion

Railside is not a perfect place - the teachers would like to achieve more in terms of student achievement and the elimination of inequities, and they rarely feel satisfied with the achievements they have made to date, despite the vast amounts of time they spend planning and working. But research on urban schools (Haberman, 1991) and the experiences of mathematics students in particular tells us that the achievements at Railside are extremely unusual. There were many features of the approach at Railside that combined to produce important results. Not only did the students achieve at significantly higher levels, but the differences in attainment between students of different ethnic groups were reduced in all cases and disappeared in some.

In this paper we have attempted to convey the work of the teachers in bringing about the reduction in inequalities as well as general high achievement among students. In doing so we hope also to have given a sense of the complexity of the relational and equitable system that the teachers implemented. People who have heard about the achievements of Railside have asked for the curriculum so that they may use it, but whilst the curriculum plays a part in what is achieved at the school it is only one part of a complex, interconnected system. At the heart of this system is the work of the teachers, and the numerous different equitable practices in which they engaged. The Railside students learned through their mathematical work that alternate and multidimensional solutions were important, which led them to value the contributions of the people offering such ideas. This was particularly important at Railside as the classrooms were multicultural and multilingual. It is commonly believed that students will learn respect for different people and cultures if they have discussions about such issues or read diverse forms of literature in English or Social Studies classes. We propose that all subjects have something to contribute in the promotion of equity and that mathematics, often regarded as the most abstract subject removed from responsibilities of cultural or social awareness, has an important contribution to make. The discussions at Railside were often abstract mathematical discussions and the students did not learn mathematics through special materials that were sensitive to issues of gender, culture, or class. But through their mathematical work, the Railside students learned to appreciate the different ways that students saw mathematics problems and learned to value the contribution of different methods, perspectives, representations, partial ideas and even incorrect ideas as they worked to solve problems. As the classrooms became more multidimensional, students learned to appreciate and value the insights of a wider group of students from different cultures and circumstances.

The role of multidimensionality in the promotion of equity is not one that has reached the attention of many researchers in the US. Culturally sensitive materials, on the other hand, have been researched in a number of different settings with some growing consensus on their effectiveness in the promotion of equity (Lee, 1995; Tharp & Gallimore, 1988). But such materials are not widely used in classrooms and can be uncomfortable for teachers if they require cultural knowledge that they do not possess or their classrooms are extremely diverse. Multidimensionality is encouraged by open curriculum materials that allow students to work in different ways and bring different strengths to their work. The use of open materials in mixed ability classrooms is something Boaler (2002a) also found to promote equity in her study of English schools. Freedman, Delp & Crawford (2005) also noted many aspects of a teachers' work that promoted equity and that are consistent with our findings, including: learners being taught to be responsible for their own learning, a learning community that appreciated diverse contributions, opportunities for different ways of learning and high challenges for all students. In Freedman et al.'s study they also found that equitable teaching did not rely on culturally sensitive materials, nor on the group work that the teachers in our study used, reminding us that there are may different routes to equity. In our study we found that mathematical materials and associated teaching practices that encouraged students to work in many different ways, supporting the contributions of all students, not only resulted in high and equitable attainment, but promoted respect and sensitivity among students.

The mathematical success shared by many students at Railside gave them access to mathematical careers, higher-level jobs and more secure financial futures. The fact that the teachers were able to achieve this through a multidimensional, reform-oriented approach at a time in California when unidimensional mathematics work and narrow test performance was all that was valued (Becker & Jacob, 2000) may give other teachers hope that working for equity and mathematical understanding against the constraints the system provides is both possible and worthwhile.

References

Becker, J., & Jacob, B. (2000, March). California school mathematics politics: The anti-reform of 1997-1999. *Phi Delta Kappan,* 529-537.

Black, P., & Wiliam, D. (1998, October). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan,* 139-148.

Boaler, J. (1997). *Experiencing school mathematics: Teaching styles, sex and setting*. Buckingham: Open University Press.

Boaler, J. (2000). Mathematics from another world: Traditional communities and the alienation of learners. *Journal of Mathematical Behavior, 18*(4), 379-397.

Boaler, J. (2002a). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning.* (Revised and expanded edition.) Mahwah, NJ: Lawrence Erlbaum Associates.

Boaler, J. (2002b). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education*, *33*, 239-258.

Boaler, J. (2002c). The development of disciplinary relationships: Knowledge, practice and identity in mathematics classrooms. *For the Learning of Mathematics, 22*(1), 42-47.

Boaler, J. (2004). Promoting equity in mathematics classrooms—Successful teaching practices and their impact on student learning. *Plenary presentation at the 10th International Congress on Mathematics Education,* Copenhagen, Denmark.

Boaler, J. (2006). Promoting Respectful Learning. *Educational Leadership*, February (in print).

Boaler, J., & Brodie, K. (2004). The importance of depth and breadth in the analysis of teaching: A framework for analysing teacher questions. In the *Proceedings of the 26th Meeting of the North America Chapter of the International Group for the Psychology of Mathematics Education*. Toronto, Ontario.

Boaler, J. & Gresalfi, M. S. (in preparation). What is learning? The importance and nature of disciplinary relationships in mathematics.

Briars, D. & Resnick, L. (2000). *Standards, assessments—and what else? The essential elements of standards-based school improvement.* Unpublished manuscript.

Chazan, D. (2000). *Beyond formulas in mathematics and teaching: Dynamics of the high school algebra classroom*. New York: Teachers College Press.

Cohen, D. & Ball, D. L. (2001, September). Making change: Instruction and its improvement. *Phi Delta Kappan*, 73-77.

Cohen, E. (1994). *Designing groupwork*. New York: Teachers College Press.

Cohen, E., & Lotan, R. (Eds.). (1997). *Working for equity in heterogeneous classrooms: Sociological theory in practice*. New York: Teachers College Press.

Corbett, D., & Wilson, B. (1995). Make a difference with, not for, students. *Educational Researcher*, *24,* 12-17.

Darling-Hammond, L. (1998). Teachers and teaching: Testing policy hypotheses from a National Commission Report. *Educational Researcher, 27*(1), 5-15.

Delpit, L. (1988). The silenced dialogue: Power and pedagogy in education other people's children. *Harvard Educational Review, 58*(3), 280-298.

Eisenhart, K. (2002) Building theories from case study research. In Huberman, A. M. & Miles, M. (Eds). *The qualitative researcher's companion*, (pp. 3-36). Thousand Oaks, CA: Sage Publications.

Fendel, D., Resek, D., Alper, L., & Fraser, S. (2003). *Interactive mathematics program: Integrated high school mathematics*. Berkeley, CA: Key Curriculum Press.

Fiori, N. & Boaler, J. (2004). What discussions teach us about mathematical understanding: Exploring and assessing students' mathematical work in classrooms. In the *Proceedings of the 26th Meeting of the North America Chapter of the International Group for the Psychology of Mathematics Education*. Toronto, Ontario.

Freedman, S. W., Delp, V. & Crawford, S.M. (2005). Teaching English in untracked classrooms. *Research in the teaching of English, 40*(1), 62-126.

Geertz, C. (2000). *The interpretation of cultures.* New York: Basic Books.

Glesne, C., & Peshkin, A. (1992). *Becoming qualitative researchers*. White Plains, NY: Longman.

Greeno, J. G., & MMAP (1997). Theories and practices of thinking and learning to think. *American Journal of Education*, *106*, 85-126.

Gutiérrez, K. (1995). Unpacking academic discourse. *Discourse Processes, 19*(1), 21-37.

Gutiérrez, K., Larson, J. & Kreuter, B. (1995). Cultural tensions in the scripted classroom: The value of the subjugated perspective. *Urban Education, 29*(4), 410-442.

Gutiérrez, K., Baquedano-Lopez, P., & Tejeda, C. (1999). Rethinking diversity: Hybridity and hybrid language practices in the third space. *Mind, Culture, & Activity: An International Journal*, 6 (4), 286-30.

Gutiérrez, R. (1996). Practices, beliefs and cultures of high school mathematics departments: Understanding their influence on student advancement. *Journal of Curriculum Studies, 28*(5), 495-529.

Gutiérrez, R. (1999). Advancing urban Latina/o youth in mathematics: Lessons from an effective high school mathematics department. *Urban Review, 31*(3), 263-281.

Gutiérrez , R. (2000). Is the multiculturalization of mathematics doing us more harm than good? In R. Malalingham & C. McCarthy (Eds.), *Multicultural Curriculum: New Directions for Social Theory, Practice and Policy* (pp. 119-219). New York: Routledge.

Haberman, M. (1991) The pedagogy of poverty versus good teaching, *Phi Delta Kappan*, 73, 290-294.

Hand, V. (2003). *Reframing participation: Meaningful mathematical activity in diverse classrooms*. Unpublished dissertation, Stanford University, Stanford, CA.

Heath, S. B., (1983). *Ways with words: Language, life, and work in communities and classrooms.* New York: McGraw-Hill; Oxford University Press.

Holland, D., Lachiotte, W., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds*. Cambridge MA: Harvard University Press

Horn, I.S. (2002). *Learning on the job: Mathematics teachers' professional development in the contexts of high school reform*. Unpublished Ph.D. Dissertation, University of California, Berkeley.

Horn, I.S. (2005). Learning on the job: A situated account of teacher learning in two high school mathematics departments. *Cognition & Instruction, 23*(2), 207-236.

Knapp, M.S., Shields, P.M., & Turnbull, B.J. (1992). *Academic challenge for the children of poverty. Summary report.* Washington, DC: U.S. Department of Education.

Ladson-Billings, G. (1994). *Dreamkeepers: Successful teachers of African-American children.* San Francisco, CA: Jossey-Bass Publishers.

Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. *American Educational Research Journal, 32*(3), 465-491.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven: Yale University Press.

Lee, C. (1995). A culturally based cognitive apprenticeship: Teaching African American high school students skills in literary interpretation. *Reading Research Quarterly, 30*(4), 608-630.

Linchevski, L. & Kutscher, B. (1998). Tell me with whom you're learning, and I'll tell you how much you've learned: Mixed-ability versus same-ability grouping in mathematics. *Journal for Research in Mathematics Education, 29*(5)*,* 533-554.

Lubienski, S. (2000). Problem solving as a means towards mathematics for all: An exploratory look through the class lens. *Journal for Research in Mathematics Education, 31,* 454-482.

Madison, B. L. & Hart, T. A. (1990). *A challenge of numbers: People in the mathematical sciences.* Washington, DC: National Academy Press.

Martino, A. & Maher, C. (1999). Teacher questioning to promote justification and generalization in mathematics: What research practice has taught us. *Journal of Mathematical Behavior, 18*(1), 52-78.

McLaughlin, M. & Talbert, J. (2001). *Professional communities and the work of high school teaching.* Chicago: University of Chicago Press.

Moses, B., & Cobb, C. (2001). *Radical equations: Math literacy and civil rights*. Boston, MA: Beacon Press.

Oakes, J. (1985). *Keeping track: How schools structure inequality*. New Haven, CT: Yale University Press.

Picciotto, H. (1995). Lab Gear Activities for Algebra 1. Aslip, IL: Creative Publications.

RAND (2002, October). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education* (Dru-2773-OERI). Arlington, VA: RAND Education & Science and Technology Policy Institute.

Rosen, L. (2001). Myth making and moral order in a debate on mathematics education policy. In M. Sutton & B. A. U. Levinson (Eds.), *Policy as practice: Toward a comparative* *sociocultural analysis of educational policy* (Vol. 1, pp. 295-316). Westport: Ablex Publishing.

Rosenholtz, S. J., & Wilson, B. (1980). The effect of classroom structure on shared perceptions of ability. *American Educational Research Journal*, *17*, 175-182.

Sallee, T., Kysh, J., Kasimatis, E., & Hoey, B. (2000). *College preparatory mathematics.* Sacramento, CA: CPM Educational Program.

Schoenfeld, A. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. *Educational Researcher, 31*(1), 13-25.

Silver, E. A., Smith, M. S., & Nelson, B. S. (1995). The QUASAR project: Equity concerns meet mathematics education reform in the middle school. In E. Fennema, W. Secada & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 9-56). New York: Cambridge University Press.

Simpson, C. (1981). Classroom structure and the organization of ability. *Sociology of Education, 54*, 120-132.

Siskin, L. S. (1994). *Realms of knowledge: Academic departments in secondary schools.* Washington, DC: Falmer Press.

Stein, M. K., Smith, M., Henningsen, M., & Silver, E. (2000). *Implementing standards-based mathematics instruction: A case book for professional development.* New York: Teachers College Press.

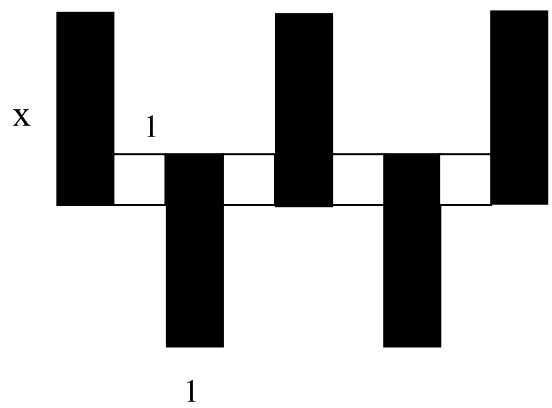
Talbert, J. & McLaughlin, M. (1996). Teacher professionalism in local school contexts. In Goodson & Hargreaves (Eds.), *Teachers' professional lives* (pp. 127-153). Washington, DC: Falmer Press.

Tharp, R. & Gallimore, R. (1988). *Rousing minds to life: Teaching, learning, and schooling in social context.* New York: Cambridge University Press.

Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Yin, R. K. (1994). *Case study research: Design and methods*. Thousand Oaks, CA: Sage Publications.

Appendix A: Groupworthy Task



Explanation of figure:
There are two types of tiles used to create the above configuration. The dark tiles are x by 1 in dimension. The light square tiles are 1 by 1 in dimension.

Task prompt:
Build the arrangement of LabGear™ blocks (shown in the diagram given to students), and find the perimeter of the arrangement.

Result (which students derive in groups):
The perimeter is $10x + 10$

Teacher follow-up question as she moves from group to group:
Where's the 10 in the $10x + 10$?

Students must discuss "where" the 10 is, and all students must be able to explain this to the teacher.

Endnotes:

---

[1] This work is sponsored by a grant from the National Science Foundation *(Division of Research, Evaluation and Communication, REC 9985146)*. The views expressed herein are those of the authors and not those of the funding agency.

[2] In the remainder of this paper we combine the students from Greendale and Hilltop that followed the traditional curriculum.

[3] In both cases we only include students who gave permission to be in the study, approximately 87% of the eligible students.

[4] This percentage includes all seniors at Greendale and Hilltop, whether they attended the 'traditional' or IMP classes. At this time we have been unable to separate the traditional students from IMP but as they were few in number this will not affect the reported percentage greatly.

[5] Greendale had one 52-minute and two 110-minute periods per week. Hilltop had three 55-minute and one 100-minute period per week.