Function Approach to Remedial Algebra
&
Implementation Through

◄ Data Relationships ►
◄ Numeric/Graphic/Symbolic ►
◄ Function Behaviors ►
&
◄ Behavior/Parameter Connection ►

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Contents adapted from:
Laughbaum, E. D. (2003). *Foundations for college mathematics*. Red Bank Publishing, Galena, OH.

Introduction

Teaching remedial/developmental algebra using a function approach can certainly be challenging since almost no textbooks use a function approach. Those claiming to use a function approach move the chapter on functions closer to the beginning or put a calculator icon next to some exercises. However, this is nearly meaningless, as functions are not mentioned (or only superficially) in other material. Further, just teaching the representations of functions early is not teaching from a function approach! Once students can freely move from numeric to graphic to symbolic, and understand the connection of all these to real world situations, then teachers can "use" function and function representation in the teaching of more traditional topics. That is, teaching from a function approach means using functions to teach mathematics, you also do and explain mathematics such as factoring, equation solving, systems of equations, inequalities, properties of inequalities, definitions, etc using functions. Using the various features of functions can enhance the teaching of almost every topic. This document is intended to provide ancillary materials for you to use while teaching a 12 – 13 day introduction to functions. If you use the argument that you do not have time for this, think again. Once your students know and understand functions you will reduce the time needed to teach many traditional topics – such as factoring, and equation & inequality solving. Further, many topics can be taught outside the classroom by using discovery activities. Thus, class time is minimized for these topics.

Aligned with "a function approach" is <u>teaching in context and teaching for understanding</u>. For most people, understanding mathematics requires investigations into the concepts underlying mathematics. Understanding mathematics is totally different from memorizing mathematical algorithms -- a series of precisely prescribed steps for solving specific types of problems, -- or "knowing" a series of facts. Many of the mathematical algorithms that are useful in the applied fields and within mathematics are available on calculators and computers; thus, the need for memorizing the algorithms has diminished. However, the need for *understanding* mathematics has increased dramatically because of the need to utilize this technology and interpret the results.

As you can see in the activities in this document, they almost always are set in a context. The reason for this is that in doing so, students can use the context to help understand the mathematics. And this is what teaching mathematics is all about – having students understand mathematics. When students understand mathematics, the related skills we teach make more sense and are easier to remember. While teaching rote may work fine in the short term, almost all is lost from memory after the midterm – to say nothing about retention in subsequent courses.

Richard P. Feynman was a Nobel Laureate in physics, as I recall, and in his 1985 book *"Surely You're Joking Mr. Feynman!"* he makes the following observation: "I don't know what's the matter with people: they don't learn by understanding; they learn by some other way – by rote, or something. Their knowledge is so fragile! … this kind of fragility is, in fact, fairly common, even with more learned people." So, learning through memorizing is, in effect, not learning.

Referencing education in Brazil, 40 or so years ago, Feynman made the following observations, "After a lot of investigation, I finally figured out that the students had memorized everything, but they didn't know what anything meant." … "Everything was entirely memorized, yet nothing had been translated into meaningful words." … It was pitiful! All the work they did, intelligent people, but they got themselves into this funny state of mind, this strange kind of self-propagating "education" which is meaningless, utterly meaningless!"

It is this idea you need to have in mind as you teach – students must understand the mathematics being taught. We are interested in our students understanding mathematics, as opposed to just memorizing algorithms. We can accomplish this goal by teaching algebra through using a function approach and all that is possible through this approach.

Methods

The primary mathematical content used to implement the use of functions in the teaching of remedial/developmental students is a beginning unit on data relationships, numeric, graphic, and symbolic representation connections, function behaviors, and behavior/parameter connections. This content, properly taught, will prepare your students for most curricula. Except for the fact that other curricula will not necessarily be "friendly" to you. That is, most curricula were developed using an "equation-solving" and "memorizing" approach. So, you will not see functions being used to factor, solve equations and inequalities, develop models of data, confirm symbol manipulations, or develop explanations of mathematical properties, laws, and definitions.

You may need to supplement with activities. You most certainly will need to change the way you teach each topic by using one or more representation of function to explain a concept, use an algorithm, etc.

Below is a brief description of what data relationships, numeric, graphic, and symbolic representation connections, function behaviors, and behavior/parameter connections will do for your students.

Data Relationships

Data relationships found in the world outside the math classroom are a natural way of introducing functions. They are numeric in representation and this helps the remedial student make a connection to the mathematics they are most familiar – numbers. Using hand-held technology to graph the data relationships as scatter plots, your students will discover that relationships fall into classes – linear, quadratic, absolute value, exponential, etc by shape. Given a relationship, they should be able to identify its shape by name. You can also choose to start function behavior analysis at this level. For example, you may want to introduce, in an intuitive way, the concepts of increasing, decreasing, maximum, and minimum using the data relationships in numeric and graphic forms.

Numeric, Graphic, and Symbolic Representation Connections

Having used 1 – 2 days teaching data relationships, looking both at the numeric and graphic representations and their connection, students are now ready for finding the symbolic representations of linear, exponential, and quadratic functions. The techniques used are described later in this document. Since the data relationships were displayed using the STAT editor on the calculator, development of symbolic form is also accomplished using the same editor. Creating symbolic form of a relationship may take 2 – 3 days.

Function Behaviors, and Behavior/Parameter Connections
This topic requires the most number of teaching hours because functions have several behaviors. They are: increasing/decreasing, maximum/minimum, rate of change, zeros, when positive or negative, and domain/range. What you will be asking of your students is to develop a basic understanding of these behaviors. Full understanding will be expected when they are re-visited in the textbook chapters. Further, after teaching a particular behavior, you may need to identify what parameter(s) in the symbolic representation of the function class controls this behavior. For example, in a linear relationship such as $dx + e$, the parameter d controls the increasing/decreasing behavior. The parameter e controls the initial condition behavior. In the symbolic representation of the quadratic function $d(x + e)^2 + f$, the parameters e and f control the maximum or minimum point – for example. At the same time you are teaching the behavior, you are also teaching what function parameter controls it – as appropriate. The behaviors of zeros and when positive or negative, are a little more complicated and the calculator is used to find them and connect them to parameters when possible.

This document also contains a variety of formative assessments, teaching tools, and activities that can be used to help students understand mathematics, and they use function as a central theme. Below is a brief description of each.

Explorations
Explorations are of two types. A few are designed for the day before a topic is discussed in class. They are somewhat like guided-discovery exercises. Just like some of the explorations are designed for before you formally teach the related topic, others have been designed for after the formal discussion. If you assign them at the wrong time, some will become exceedingly difficult and others may take what was a challenging assignment and turn it into a simpler and low-level thinking task. You must look at each exploration and decide when to use it. An exploration that asks students to predict the number of inmates in the US in the year 2005, given a numeric history of the prison population, is assigned before you actually teach this topic. The intent is to see if they can use their own ideas to solve this problem. Most explorations require a graphing calculator, a few don't. The author used almost all of the explorations as group assignments; giving them to students as they left a class and collecting them at the beginning of the next class. The title "Explorations" is somewhat descriptive of the kind of work the students will be doing. That is, many times students must explore on the calculator to answer the question. <u>Explorations may be used to teach a behavior, or to have students discover a parameter behavior connection.</u>

Assessment tools like the explorations measure a totally different kind of learning than do skill-based midterms. They will require students anywhere from

10 minutes to an hour to finish – depending on the student and on the exploration.

Concept Quizzes

Just like the explorations, some concept quizzes are to be assigned either before a topic is discussed or right after. Many of these are like guided discovery exercises and they would be assigned before you teach the topic in class. What is unique about the concept quizzes is that many ask students to do something they probably don't do much – be creative. These quizzes contain questions like "If $(f+g)(x)=2x+7$, develop any two functions $f(x)$ and $g(x)$ whose sum is $(f+g)(x)$." Or "Create any function that has a domain of [–3.6, ∞)." The concept quizzes usually require a graphing calculator.

The concept quizzes are an integral part of the overall assessment tools. You may want to consider making them as much as 10 to 15% of a student's grade. Tools like the concept quizzes measure a totally different kind of learning than do skill-based midterms and the explorations that require "exploration" and a little tenacity. Concept quizzes require students anywhere from 5 minutes to half an hour to finish – depending on the student and on the quiz.

Investigations

The investigations usually take a situation or single idea and ask a multitude of questions about the situation or idea. They are assigned after a topic has been developed in class. They usually require a graphing calculator. The intent of the investigations is to require a thorough analysis of a topic or idea. For example, one investigation asks 14 questions about the electricity charge data from the North Carolina Public Utilities Commission. Or another investigation has 26 questions about gasoline usage in a car. Of course the questions are mostly all related to the mathematics being taught. The real-world situation is used to help students understand the mathematics.

The investigations are an integral part of the overall assessment tools. Investigations require students anywhere from 10 minutes to an hour to finish – depending on the student and on the investigation.

There is no answer key for these activities. The reason is twofold. First, many of the questions are open-ended, meaning that there are many correct responses. Second, sample answers encourage students and instructors to look for answers similar to the answers in the answer key. This stifles creativity. Creating your own answer from your own mind is a highly rewarding experience and causes a desirable higher level of thinking. The spark that can cause this higher level of thinking may come from a group setting. Working toward a "back-of-book" answer impedes the process. These materials may be duplicated for face-to-face classroom use only. They may not be reproduced in any format for any other purpose without the written consent of the author.

Implementation

There may be three main problems in implementing "a function approach" to the teaching of remedial algebra. The first is a lack of materials; the second is funding for the required hand held equipment, and the third is professional development for teachers using the function approach. This document is intended to help with the first problem; however, I realize that it is not the best solution. Using a remedial textbook that uses a function approach is highly desirable and is the most reasonable solution.

As for the equipment issue, it is necessary for students to have the calculator with them at all times they may be working on homework or are in class. This implies that students must purchase their own, or your school needs to have a loaner program whereby each student is loaned a calculator for the duration of the course. Another option that is somewhat popular is to provide a rental program. Under this approach, the school owns the calculators and rents them to the students in need for a nominal fee of $20 or so per semester. A mixture of these models may be the best alternative.

The initial purchase of the equipment can be problematic for the teacher wanting to move forward with mathematics education at his or her school. You must present your case to the administration with sufficient rationale to be convincing. This must include references and research on the use of hand held technology in the teaching of mathematics. For this, please read the Executive Summary of *Handheld Graphing Technology at the Secondary Level: Research Findings and Implementations for Classroom Practice*. The full book is available by request at http://education.ti.com/global/promo/research.html, as is the Executive Summary. You may also find a pre-print of the article "Algebra with Function as the Underlying Theme" by Laughbaum at www.empt.org useful. Another source of convincing testing data can be found at www.empt.org starting in the late fall of 2002. You may further argue that since the graphing calculator is required by 50% – 75% of the college mathematics courses in the country – see http://www.math.ohio-state.edu/~elaughba/chapters/98survey.pdf and www.ams.org/cbms/, and the function approach requires a graphing calculator, that you will be better preparing your students for college. At the same time, teaching for understanding will better prepare your students for life and work.

Once administrative approval has been obtained, the next problem is procuring funding for the equipment. One source of ideas is found at http://education.ti.com/t3/resources/artfunding.html. Basically, this page suggests a local approach. That is, try to find local businesses and/or charities to fund your project. The page also lists several national foundations, but applying to a national funding agent may take too long. So, if you will be using the rental program mentioned above, the monies found through local businesses and industry will purchase the first few sets of equipment. Once purchased, they will be self-sustaining through the equipment rentals.

Professional development for your faculty is provided through the EMPT Program. As of the summer of 2002, the only option is the EMPT Summer Academy. We are expecting a few regional professional development centers to offer one-day workshops throughout the 2002-2003 academic year, but the schedule and agenda has not been set. Your school may also apply for an academic year workshop at www.empt.org.

If you need help with the graphing calculator, you may want to try the web site http://www.prenhall.com/divisions/esm/app/calculator/ for a free web-based tutorial. You might also consider going to education.ti.com and look at their on-line course on algebra using a graphing calculator. Texas Instruments also has a video series called "Algebra with the TI-83 Plus." It may provide you with help. Another possibility for calculator help is to become a VIP User. This is a TI program that gives you free access to many of their calculator apps and something called watch me ware. You will also find basic calculator tutorials at http://www.esc4.net/math/.

Caution

Changing your teaching to the philosophy behind "a function approach" is complicated. Probably all of your educational experiences, your training in college, and your teaching experience have been based on the philosophy of the "equation-solving" approach. The way you think about mathematics, the way you teach mathematics, and the expectations you have of your students all require a change. Change takes work on your part. You must re-think everything you do, what you say, how you say it, when you say it, when you do it, what you assign, why you assign it, how you answer questions, how you use technology, when you use technology, when you expect your students to use technology, your conversations with colleagues, what you look for in professional development, etc. Good luck and make good decisions.

Also be reminded that this is an introduction to teaching algebra from a function approach. As such, you do not "cover" completely all of each function or each behavior to be taught. Keep concepts intuitive. Your goal is to develop an understanding of ideas and content, not to develop a mastery of concepts and skills. Mastery of concepts and skills comes later in the course when functions are taught, along with more traditional algebra, in a full chapter dedicated to each function type.

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Laughbaum, E. D. (2003). *Foundations for college mathematics*. Galena, OH. Red Bank Publishing

Laughbaum, E. D. (2000). *Explorations, concept quizzes, investigations, writing mathematics, modeling projects for foundations for college mathematics*. Galena, OH. Red Bank Publishing

Background Information and “Expert” Opinion

Function Approach

Email from Robin Rider on her dissertation research:
What I have done is compare two developmental math programs, one that teaches from a function approach, using multiple representations (MR) and graphics calculators. The other is a traditional university algebraic, no calculator approach. I gave them pre and post-tests. They could solve the problem using any method they wished. Hypothetically, both programs, which have the same concepts taught, should have been equally successful. However, what I have found is that not only were the students in the function, MR curriculum more successful (indicated by a higher percentage of correct problems on the post test), there were a large percentage who used more than one representation to solve the problem (or to check), thus their flexibility with the different representation was increased.

I am writing a paper for a conference in July on my research, I will be happy to send it to you with the results when I am finished. Robin Rider, Assistant Director, CSMTE, East Carolina University (April 16, 2003)

Laughbaum, E. D. (2003). *MATHEMATICS AND COMPUTER EDUCATION, 34(1), 63-71.*
“The concept of function is one of the central ideas of pure and applied mathematics. For nearly a century, recommendations for school curricula have urged reorganization of school mathematics so that the study of functions is a central theme. Computers and graphing calculators now make it easy to produce tables and graphs for functions, to construct formulas for functions that model patterns in experimental data, and to perform algebraic operations on functions.”

Chapter Five, page 42. Recommendations for High School Teacher Preparation. Conference Board of the Mathematical Sciences. 2001. *The Mathematical Education of Teachers*. American Mathematical Society. Washington D. C.

Susan E. Williams from the University of Houston says in her article “Effects of Hand-Held Computers on the Teaching of Algebra,” <www.coe.uh.edu/insite/elec_pubHTML1997/ma_will.htm> (12/07/00) “The publication, ‘Algebra for Everyone’ (Edwards, 1990), suggested that all high school students need to study algebra. Several mathematics educators supporting this idea have noted that traditional algebra is approached in a manner that is too abstract for most students (Chambers, 1994; Hawkins, 1993; Heid, 1995; National Council of Teachers of Mathematics Board, 1993; Seeley, 1993; Silver, 1995; Steen, 1992; Usiskin, 1995). A recurring message in each of these articles is that traditional algebra often is taught using a rigorous approach that involves rote computations, meaningless manipulations of symbols, and acquisition of a predefined set of procedures for solving a fixed set of contrived problems.” She continues to say, “One of the major themes that permeates mathematics in algebra and the courses that follow is the study of function. In spite of this, traditional algebra has been organized around the concept of equation and methods of solving equations. The concept of function has been ‘patched in’ at the end of the algebra course without providing substantial meaning or purpose.”

It has long been the observation of the author that we rarely had much success in developmental algebra using a traditional curriculum and approach (Laughbaum, 1992). Generally speaking, students take remedial courses not knowing many of the basic manipulation skills upon entering and when they leave our courses, they still do not know many of the basic manipulation skills. So, what is gained? Textbooks, and faculty not wanting to change drive the curriculum. Textbooks almost always use an equation approach and emphasize symbol manipulations. Using a function approach is a viable option for making a difference in the developmental curriculum.

"Many teachers and researchers know that the presentation of algebra almost exclusively as the study of expressions and equations can pose serious obstacles in the process of effective and meaningful learning."

Kieran, C. 1992. "The Learning and Teaching of School Algebra." In *Handbook of Research on Mathematics Teaching and Learning*, edited by Grouws, D. A. pp. 390-419. Macmillan. New York.

Use of the Graphing Calculator

Keynote Address, T³ Regional Conference, Columbus, OH, March, 2003
Charles Vonder Embse, Department of Mathematics, Central Michigan University

Students who use graphing calculators:

- Have a better understanding of the concept of functions;
- Exhibit dynamic notions of functions;
- Prefer graphical representations;
- Are better able to represent the whole graph based on a partial graph.
- Have a deep understanding of the concept of variable;
- Are better able to solve algebraic representations of realistic problems;
- Have a better understanding of graphical representations;
- Have a better understanding of applications.
- Have equivalent procedural skills.

Critical Factors for Learning

- Simple introduction of graphing calculators into the classroom is not enough;
- Greater access to the technology means greater impact on student learning;
- Must have a curriculum that supports the technology tools;
- Student and teacher interactions;
- Students' existing mathematical knowledge and beliefs.

Ohio Early College Mathematics Placement Testing Program Data:

Figure 1: Average scores for juniors in the EMPT-test (1997-2001)

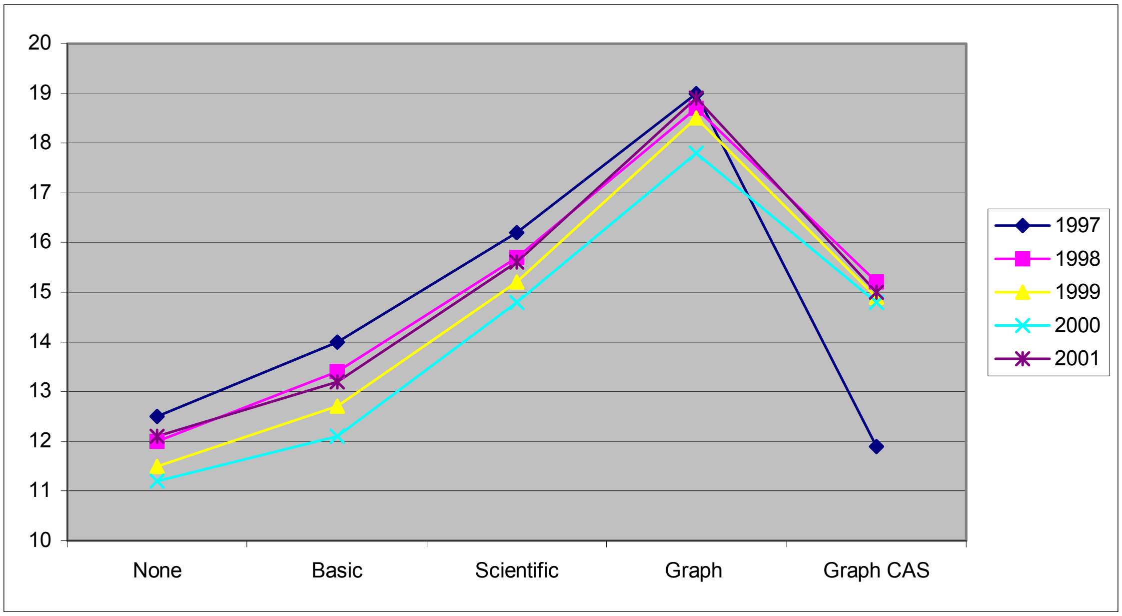
26-32 places in pre-calc

18-25 places in college algebra

12-17 places in intermediate algebra

0-11 places in arithmetic or beginning algebra

State mean = 16.5



Those with no calculator were clearly scoring less in the test than those with a calculator. Students using a graphing calculator without CAS scored the best average scores having a 6-point higher performance than those without a calculator. This is one full placement level higher.

The Ohio State University Mathematics Placement Test & Graphing Calculators (www.empt.org)

The study led to the following conclusions:

- On the Math B Test, there is good statistical evidence that the graphing calculator gives students with ACTs of 21 or below an advantage of 2-4 questions.
- On the Math B Test, the data suggests that students with ACTs of 22-24 who use the graphing calculator may have a smaller advantage, of about 1 question.
- On the Math D Test, there is statistical evidence that the graphing calculator gives students with ACTs of 23-26 an advantage of 2-4 questions.
- On the Math D Test, the data suggests that students with ACTs of 27 or more who use the graphing calculator may have a smaller advantage, of 1 question or more.

These conclusions, based on the data, indicate that there are many students who place at a higher Course Code on the placement exam when using the graphing calculator than they would have if they had not been permitted to use it. The question arose as to whether these students would, as a result of using the graphing calculator, be harmed academically by being placed in the higher math course. In order to answer this question, all students from the study were tracked to find the grades in their Autumn 96 courses and make the same comparisons. If there were a significant disadvantage to the students who had achieved their placement result using a graphing calculator, then those students would have received lower grades when compared to students in the same class who had not used the graphing calculator on the placement exam. The data shows that, in general, the average grades of the two groups were close to each other in most classes. Of those groups of students in which the scientific calculator students had higher grade averages than the graphing calculator students, none of the differences were statistically significant. The one anomalous result was for the students who placed into Math 075 (Intermediate Algebra); in this group, the graphing calculator students performed significantly better for no discernible reason.

The report concludes that: "So far, the results indicate that the use of the graphing calculator does produce slightly better placement scores, but so far has not been shown to put the students at any disadvantage in the classes they place into."

Mark Garner & Michelle Fawcett,
The Ohio State University
Department of Mathematics

Note: This article was written in 1997. The Ohio State University Mathematics placement tests are now on-line, but the above results have not changed.

Equation/Function Comparison

When using a traditional algebra text/curriculum and supplementing it with graphing calculator activities or simply doing the mathematics with a graphing calculator, the assumption is that students may think of the calculator as a black box and not really understand what it is doing or how it is related to the mathematics being taught. Students may not understand the connection between a graph or table and the related mathematics being performed.

Further, the authors of graphing calculator activities usually have a mindset based on a traditional curriculum, so their activities often reflect the use of technology as applied to the "equation solving" approach.

On the next page is an example of an activity from a person who is strong on calculator use and teaching the traditional curriculum. The activity appears to be related to "linear equations." He asks students to find the change in the average distances from year to year, but ignores the concept of rate of change. He then asks students to plot the data to see if suggests that it might be linear. He then directs students to find a linear regression model, but does not connect the results to the change they found earlier. Since the audience is beginning algebra, one must wonder if they know anything about regression equations. Once students have the model, they are asked to use it to make some predictions. He finishes with asking students to replace calendar years with generic time and then asks how the regression equations are related, but does not discuss a the geometric transformation of a horizontal shift.

Ok, some good stuff, but lots of mathematics missing as related to the ideas behind a linear function in particular and functions in general.

From an Equation Solving Approach

Sex Drives - The Average driving distances on the men's and women's tours New York Times Magazine, May 25, 2003 (Data from the PGA and the LPGA)

Question 1

(a) The table given below shows the average driving distance (in yards) for players on the men's and women's tours. Mentally estimate the change from one year to the next (the finite differences) for each set of data and decide if the growth appears to be linear or non-linear for either case.

Year	PGA	LPGA
1992	260.5	223.2
1993	260.4	226.3
1994	261.8	226.6
1995	263.6	233.4
1996	266.5	232.9
1997	267.7	236.3
1998	270.6	236.7
1999	272.4	238.1
2000	273.2	238.4
2001	279.4	242.4
2002	279.8	248.1
2003	285.2	251.5

(b) Enter the data from the table into the List Editor of a TI83. Plot the graphs of the distances for each tour versus time and compare the results to your thoughts from part (a).

(c) Find the line of best fit for the two sets of data plotted in part (b). Is a linear model appropriate for these sets of data? If it is not, suggest and find another model.

(d) Use your regression equations from part (c) to answer the following questions. You may assume that the trends you have observed continue into the future.

(i) Estimate the average driving distances for both tours in the year 2005.
(ii) When will the average driving distance for the men's tour be 300 yards?
(iii) When will the average driving distance for the women's tour be 260 yards?
(iv) When will the average driving distance for the women's tour be higher than the average for the men's tour?

(e) Replace each year in the table by a number from 0 to 11, with 0 representing the year 1992 and 11 representing the last year, 2003. Find the regression equations and compare them to those you found in part (c). What aspects of the equations remained the same if any?

From a Function Approach

Sex Drives - The Average driving distances on the men's and women's tours
New York Times Magazine, May 25, 2003 (Data from the PGA and the LPGA)

The table given below shows the relationship between time and average driving distance (in yards) for players on the men's and women's golf tours.

Year	*Time*	*PGA*	*LPGA*
1992	0	260.5	223.2
1993	1	260.4	226.3
1994	2	261.8	226.6
1995	3	263.6	233.4
1996	4	266.5	232.9
1997	5	267.7	236.3
1998	6	270.6	236.7
1999	7	272.4	238.1
2000	8	273.2	238.4
2001	9	279.4	242.4
2002	10	279.8	248.1
2003	11	285.2	251.5

1. After looking at the graphical representation of *Time* vs. *PGA* and then *Time* vs. *LPGA*, what general shapes do they seem to have? __________________________

2. Do the relationships appear to be increasing or decreasing? __________________________

3. The conjecture is that the data might be best modeled by a linear function. Do you agree? (If not, propose another model type, and take an "F" for this activity. ☺ __________________________

4. Find the average rate of change of drive length for the *PGA* golfers.______________________

5. What is the initial condition for the *PGA* golfers? __________________________

6. Find the average rate of change of drive length for the *LPGA* golfers.______________________

7. What is the initial condition for the *LPGA* golfers? __________________________

8. From the *Time* vs. *PGA* relationship, find the average *Time* and average *PGA* (driving distance). Express them as another data point. We will call it the "average point." __________________________

9. From the *Time* vs. *LPGA* relationship, find the average *Time* and average *LPGA* (driving distance). Express them as another data point. We will call it the "average point." __________________________

10. For either the *PGA* or *LPGA* golfers, how might the numbers found for average rate of change and initial condition help you find a model for the data?

11. For either the *PGA* or *LPGA* golfers, how might the numbers found for average rate of change and the average point help you find a model for the data?

12. Propose a model for the *Time-PGA* data relationship.

13. Do you think your model is the best possible? Explain.

14. Propose a model for the *Time-LPGA* data relationship.

15. Do you think your model is the best possible? Explain.

16. Do your models appear to go through the data points? Should they?

17. If your model for the *Time-PGA* data relationship is linear, what is the rate of change you used, and what is the initial condition of your model?

18. If your model for the *Time-LPGA* data relationship is linear, what is the rate of change you used, and what is the initial condition of your model?

19. Using your model for the *Time-PGA* data relationship, how far will the average drive be in 2004?

20. Using your model for the *Time-LPGA* data relationship, when will the average drive length be 255 yards?

21. What is a reasonable problem domain for the data relationships? Explain your reasoning.

22. How can you change your *Time-PGA* model so that the domain is [0, 20]? ____________________

____

23. If the domain of your *Time-PGA* model is [0, 20], what is the range? ____________________

24. If the domain of your *Time-PGA* model is [0, 20], does the function (model) have a maximum or minimum? If yes, what are the maximum and minimum values? ____________________

____

____

25. Using the initial data, is the average drive length increasing faster for the *PGA* or *LPGA*? Explain. ____________________

____

26. Do you think there will be a time when the average driving distances will ever be the same for men and women? Explain. ____________________

____

27. If the average drive lengths were increasing at the same rate, would the *LGPA* drive length ever catch and overtake the *PGA* length? Explain. ____________________

____

28. If one point on the graphical representation *Time-PGA* data relationship were (13, 255), explain what these numbers mean. ____________________

____

29. Give an example of other data relationship that is linear in nature. ____________________

____

30. What is one characteristic of a data relationship that helps you decide if it can best be modeled with a linear function? ____________________

Data Relationships

Data relationships abound in the world around our students. Normally, they can read and understand them. Looking at data relationships is our first step in developing algebra from a function approach. How do you convert data pairs to a graphic form? What can we learn from the numeric forms of the function? Here is where hand-held technology provides a source of the data and a display of the relationships in numeric and graphic forms. The graphing calculator can be used as a black box tool until you are ready to teach domain/range with the WINDOW feature of the calculator.

Please note that data from the world outside the algebra classroom isn't structured in the order of "difficulty." That is, students may encounter linear, exponential, or quadratic data at nearly the same time. Should we ban our students from reading the newspaper or watching the evening news because they may encounter the words constant rate, increasing, maximum, etc. before we teach function behaviors? Do we say that the biology teacher can't "cover" the growth of a bacteria population, or the physical science teacher can't "cover" projectile motion because "we" haven't gotten to exponential or quadratic functions yet? No.

If you are not teaching from a textbook that uses a function approach, your text may not use the word function, but rather, equation (except in the chapter on functions). That is, many texts teach linear equations followed by quadratic equations, etc. It is here that you will need to correct the approach and differentiate between the generic "function" and the very limited and specific "equation."

This first look at data relationships and the related function behaviors is an introduction to algebra from a function approach. As such, you do not completely "cover" all of each function or each behavior to be taught. Keep concepts intuitive. Your goal is to develop an understanding of ideas and content, not to develop a mastery of concepts and skills. Mastery of concepts and skills comes later in the course when functions are taught in a full chapter dedicated to each type.

What follows are a series of student activities that are designed to develop this intuitive sense about the concept of function and function representation connections. The first set teaches students about the connections among data relationships in the real world, their graphical representations (shapes), and a for-shadowing of function behaviors of increasing, decreasing, and average rate of change. The rate of change idea can be deleted for this set if you think your students may not be ready. Rate is extremely important in mathematics and it is addressed again later in the materials.

At this point in the development, you should use the graphing calculator as a black box and teach your students just the basics of graphing data sets. They need to know how to ungroup a group file, and run a program. (This assumes you have sent the group file to student calculators without their knowledge.) Individual programs, when executed, copy the data into L1 and L2 so that they then need to learn to set up STAT PLOT to produce the graph, followed by ZOOM STAT to get the actual graph of the data relationship. Using the numeric representation is also instructive, so students need to be able to use STAT EDIT after each program has been run.

In terms of class time, the following activities might require 2 – 3 class days of typical 50-minute classes. You are not expecting total mastery of the topic of moving between these two representations of a function. As you will recognize, we have only gone from numeric to graphic representation. We want our students to know that mathematics has a real world connection, and technology can be used to move from one representation to another. We want our students to know the real world has an order to it, and that we will be able to analyze it via mathematics. That is, given a data relationship, students must be able to categorize it by type based on shape.

Exploration Class________ Name________________________________

1. In the data relationships that follow, identify a "shape" of the graphical representation – if apparent.
2. Describe whether the relationship exhibits an increasing behavior, or decreasing behavior? Explain.
3. Does the relationship seem to be changing at a constant rate? Explain.

Below are the possible wages as provided by the local Restaurant Association for a server working at the Blue Point Café. He is paid a salary plus he earns an average of $3.50 in tips per table served. (BLUEPTDB)

Salary Data

Tables	10	15	20	25	30	35	40	45	50	55
Wages	75.00	92.50	110.00	127.50	145.00	162.50	180.00	197.50	215.00	232.50

1____

2____

3____

Below is data showing how charges from the electric company depend on the number of kilowatt-hours of electricity used per month. (ELECTRDB)

Electric Bill Data

kWh	200	400	600	800	1000	1200	1400	1600	1800	2000
Charge	24.36	40.23	56.09	71.95	87.82	103.68	119.55	135.41	151.28	167.14

1____

2____

3____

Below is the data showing the number of people enrolled in Medicare for the last few years. The numbers have been rounded to the nearest 100. (MEDICR32)

Medicare Data

Time	1975	1980	1985	1990	1991	1992	1993
Number	23,800	27,500	30,100	33,100	33,800	34,400	35,100

1____

2____

3____

Suppose you are planning on mowing grass as your summer job. Historical evidence shows that the area remaining to be mowed is related to the time mowed on a 12,000 square foot lawn. (LAWNDB)

Lawn Data

t	0	5	10	15	20	25	30	35	40
Area Remaining	12000	10500	9000	7500	6000	4500	3000	1500	0

1____

2____

3____

Below is the percent of the Unites States Gross National Product spent on health care. Time is in calendar years. (GNP24)

Health Care Data

Time	1960	1965	1970	1975	1980	1985	1990	1995
% of GNP	5.3	5.9	7.3	8.3	9.2	10.5	12.2	14

Source: U.S. Dept. of Health and Human Services

1____

2____

3____

Collect "live" data from the CBR as you walk at a constant (speed) average rate of change going away from the CBR. **(What happens if you don't walk at a constant rate?)**

Walker Data

t										
D										

1____

2____

3____

Exploration Class________ Name_________________________________

1. In the data relationships that follow, identify a "shape" of the graphical representation – if apparent.
2. Describe whether the relationship exhibits an increasing behavior, or decreasing behavior? Explain.
3. Does the relationship seem to be changing at a constant rate? Explain.

These data show the relationship between the Centigrade (Celsius) temperature and the weight of one gallon of water in pounds. (WTRWTDB)

Water Weight Data

T	0°	1°	2°	3°	4°	5°	6°	7°	8°
W	8.33461	8.33513	8.33551	8.33575	8.33585	8.33582	8.33565	8.33536	8.33495

Source: *CRC Handbook of Chemistry and Physics College Edition*. 1965-1966. The Chemical Rubber Company. Cleveland, OH

1____

2____

3____

A 12-ounce can of soda is equivalent to 354.8824 milliliters (*ml*). The volume of 354.8824 *ml* of water (soda is mostly water and will behave in a similar fashion) is measured at varying temperatures and the data is displayed in numeric form below. (POPCANDB)

POP Can Data

T	–10	–5	0	2	4	6	10	20
V	355.5425	355.1308	354.9285	354.8930	354.8824	354.8930	354.9782	355.5105

1____

2____

3____

Below is the data showing the relationship between the selling price (*s*) of a graphing calculator and the daily profit earned (*P*) from the sale of all calculators sold that day at that selling price. (CALCULDB)

Calculator Profit Data

s	50	55	60	65	70	75	80	85	90
P	15890	17540	18690	19340	19490	19140	18290	16940	15090

1____

2____

3____

What is the shape of the data relationship between the velocity (*v*) of blood in an artery that has a radius of 0.5 cm and the distance (*d*) the blood is from the center of the artery? A distance of 0.5 cm from the center means the blood is touching the wall of the artery. Velocity is in *cm*/sec. (BLOODDB)

Blood Speed Data

D	0	0.1	0.2	0.3	0.4	0.49
V	25	24	21	16	9	0.99

1____

2____

3____

Collect "live" time-height data from the CBR as a ball is tossed straight up in the air over the CBR.

Ball Pitch Data

t										
h										

1____

2____

3____

Exploration Class________ Name________________________________

1. In the data relationships that follow, identify a "shape" of the graphical representation – if apparent.
2. Describe whether the relationship exhibits an increasing behavior, or decreasing behavior? Explain.
3. Does the relationship seem to be changing at a constant rate? Explain.

As a commercial airliner flies from Columbus to Pittsburgh (approximately 200 miles), its height (in feet) is related to the time in flight from Columbus. Below is the data showing the time in the air and the related height of the plane. (PLANEDB)

Plane Data

t	0	5	10	15	20	25	30	35	40
Height	0	4500	9000	13500	18000	13500	9000	4500	0

1____

2____

3____

Below is the numeric representation of the temperature T at time t in Columbus, Ohio during an "Alberta Clipper" cold front. (CLIPP124)

Clipper Data

t	0	1	2	3	4	5	6	7	8	9	10
T	2	0	–2	–4	–6	–8	–6	–4	–2	0	2

Time (t) zero is midnight.

1____

2____

3____

Continued

The amount of land being farmed is shown below as a percentage of the total land in the United States. (FARMLNDB)

Farmland Data

Time	1850	1870	1890	1910	1930	1940	1950	1960	1970	1980	1990	2000
% Farmed	15.6	21.4	32.7	38.8	43.6	46.8	51.1	49.5	47	44.8	42.7	41.6

Source: U. S. Bureau of the Census, *Statistical History of the United States* (1970), *Statistical Abstracts of the United States.*

1____

2____

3____

Exploration Class________ Name________________________________

1. In the data relationships that follow, identify a "shape" of the graphical representation – if apparent.
2. Describe whether the relationship exhibits an increasing behavior, or decreasing behavior? Explain.
3. Does the relationship seem to be changing at a constant rate? Explain.

Below is the relationship between IQ and weight of 10 randomly selected students. (IQDB)

IQ Data

Weight	110	115	118	127	135	138	142	150	155	170
IQ	120	115	130	103	95	137	120	112	128	116

1____

2____

3____

The data below shows a checking account balance for the month of March, 1994. The time is in the day of the month and the balance is rounded to the nearest dollar. (CHECKBOO)

Dollar Data

Time	1	3	5	7	9	11	13	15	17	19	21	23	25	27
Balance	3245	3202	958	885	623	623	623	623	623	1847	551	464	198	43

Source: author's checkbook

1____

2____

3____

A server at the Blue Point Café is responsible for filling the ice chest in the cafe. He kept the following records on the number of times per day he had to fill the chest. Time is expressed in the day of the month. (ICEDB)

Ice Data

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number	6	6	8	4	12	9	3	4	6	5	8	11	6	5

1____

2____

3____

Exploration Class________ Name_________________________________

1. In the data relationships that follow, identify a "shape" of the graphical representation – if apparent.
2. Describe whether the relationship exhibits an increasing behavior, or decreasing behavior? Explain.
3. Does the relationship seem to be changing at a constant rate? Explain.

Below is the relationship between time and the national debt in the United States. (DEBTDB)

Debt Data

Year	1940	1950	1960	1970	1980	1990	1994	2002
National Debt	51	257	291	381	909	3113	4500	6000

(Debt is in billions.)

1____

2____

3____

The human population of the earth has been steadily increasing as time passes. Below is the data showing the relationship between time and population. The population is in billions. (POPULDB)

World Population Data

t (year)	1000	1750	1800	1850	1900	1950	1994	2001
P (Human Population)	0.2	0.8	1	1.2	1.7	2.8	5.4	6.2

1____

2____

3____

Below is data showing the relationship between time (*t*) in minutes and the number of E. coli bacteria (*N*) on an uncooked piece of hamburger? (ECOL5000)

E. coli Data

t	0	30	60	90	120	150	180
N	5000	14142	40000	113137	320000	905097	2560000

1____

2____

3____

Below is the data relationship between time and public education expenditures (in billions) in the United States? (EDUCDB)

Education Data

Time	1940	1950	1960	1970	1980	1990
Expenses	3.3	8.9	23.9	68.5	165.6	377.5

Source: U.S Department of Education, National Center for Educational Statistics, *Digest of Education Statistics*, 1992

1____

2____

3____

Collect "live" time-temperature data as a heated temperature probe (in your hand) cools for 30 seconds.

Cooling Temperature Data

Time	0	5	10	15	20	25	30
Temp							

1____

2____

3____

Connections: Numeric, Graphic, and Symbolic

At this point in the development of using a function approach, the students have used many sets of data that will address typical remedial (developmental) algebra content. They have seen that real world data relationships can be represented in graphic form through the use of technology and the mathematical idea of Descartes. They understand that relationships can be categorized by shape.

We are now ready to take our students to a higher cognitive level by demonstrating that these data relationships and "pictures" of data relationships can be symbolized! This is equivalent to a quantum leap for developmental (remedial) students. But, as you will see, the leap in cognitive level is not quantum when technology is used. We will lead our students through what they know and understand, number and pattern recognition, to this new level of understanding and representation.

Typically, developmental (remedial) students don't have an appreciation of why mathematics teachers are so hell-bent on teaching mathematics with all those mathematical symbols. They may think that these symbols are not connected to the real world in any way, and that the work with symbol manipulation has no meaning or connection to their lives. Further, they may just see mathematics education as a roadblock to a degree.

So we finally have a method for convincing students that the symbols (variables and operational) we use in mathematics have a real-world meaning. We will convince our students that symbols can be used to represent data relationships. That without symbols we are limited in what we can do. We will convince students that there is need for the idea of "variable" and arithmetic operational symbols.

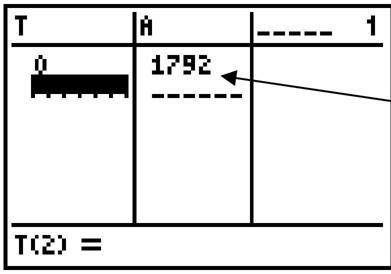
As you can see in the student activities that follow, you and your students need to be able to use selected functionality of the STAT (list) editor. You need to be able to insert and name lists, remove a list from the editor and from memory, move lists from memory to the editor, insert and/or remove new elements to/from a list, use arithmetic in the edit line, and (most importantly) use symbols to represent the elements of a list.

The activities that follow are for classroom use by the teacher as guided discussions. You will note that the activities address not just simple linear functions, but exponential and quadratic as well. Your students need to make the association that similar shapes have similar symbolic representations. They should have around 50%, mastery of creating the symbolic form of linear relationships, and considerable less for exponential and quadratic. Full mastery should be expected later in complete chapters on these functions.

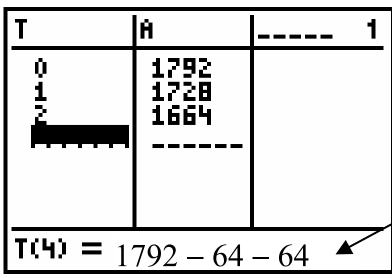
A suggested time frame for this material is 2 class days. Some may want to add a third day. This document contains no student activities, yet they are needed. It is suggested that you and your colleagues do a little group work to develop them.

Creating Symbolic Form

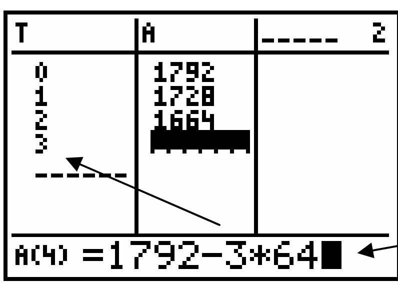
House Painter Measurements on a potential customer's house shows a surface area of 1792 square feet. Let *t* be time in hours and *A* the area that remains to be painted. Since you paint at a rate of 64 square feet per hour, and the initial conditions are known, develop the algebraic expression.



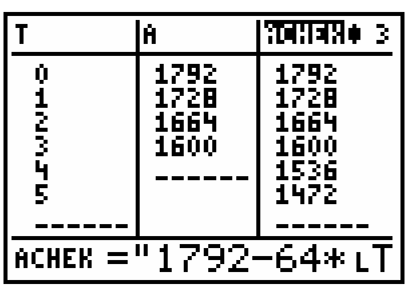
The initial conditions are known. At time 0, the area remaining to be painted is 1792 square feet.



Since the rate is known, you also know that each hour worked is another 64 square feet less to paint. The mathematical process is subtraction of 64. Thus, we can create a numeric representation from the verbal description.



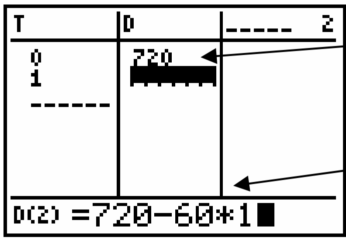
But repeated subtraction of 64 can best be accomplished by subtracting the product of 64 and the number of times it has been subtracted – as shown in the edit line.



From a pattern of arithmetic operations, you may generalize to represent the situation and numeric representation in algebraic symbols such as 1792 – 64*T*.

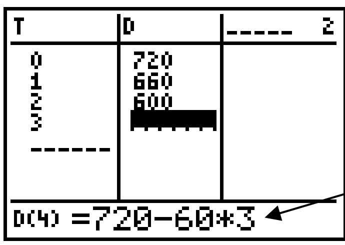
At this point, make the connection between representations. That is, graph the scatter plot of all points (T, ACHEK). Follow this with the graph of the model 1792 − 64*x* using the Y= editor. Use trace to compare the data points with points on the graph of the model. Be sure to connect the symbolic to the numeric by using the TABLE feature of the calculator. Discuss the use of −64*x* + 1792 vs. the use of 1792 − 64*x*. This gives you the opportunity to review the commutative property of addition, and the definition of subtraction.

Driving Home On the 720-mile return trip home from Kitty Hawk, North Carolina to Columbus, Ohio, Professor Ed drove at an average rate of 60 mph throughout the night. Create a mathematical model that will find the distance *D* left to travel after *T* hours driven.

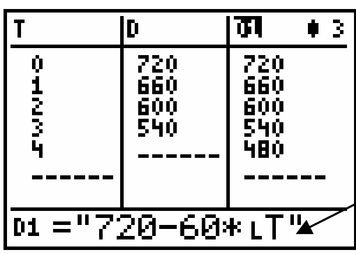


The initial condition is known.

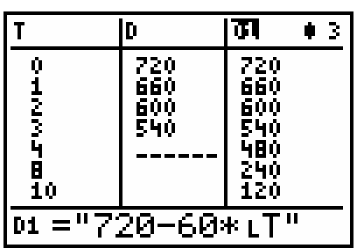
The rate is known and subtracting it for each hour driven creates the distance left to travel.



After three hours driving, most students will be able to generalize to 720 − 60∗3 rather than 720 − 60 − 60 − 60 (with your help).



From the generalized numeric form, you can go directly to the symbolic representation of the data relationship.

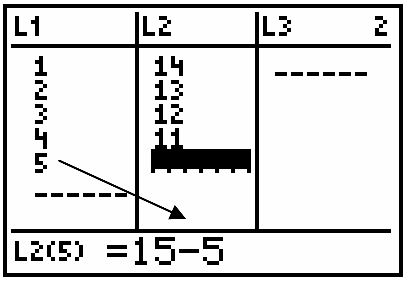


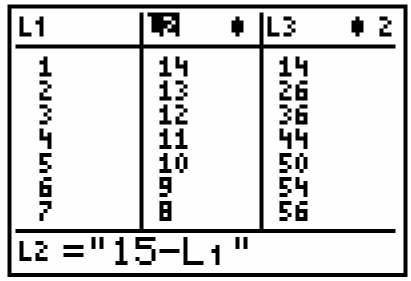
Once you have the generalized symbolic representation, include a few extra values for time so that students can see the power of knowing the symbolic form 720 − 60*T*, where *T* is the time traveled. (The ∗ sign is optional.)

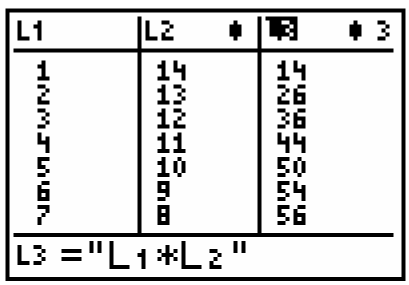
At this point, make the connection between representations. That is, graph the scatter plot of all points (T, D1). Follow this with the graph of the model 720 − 60*x* using the Y= editor. Use trace to compare the data points with points on the graph of the model. Be sure to connect the symbolic to the numeric by using the TABLE feature of the calculator. Discuss the use of −60*x* + 720 vs. the use of 720 − 60*x*. This gives you the opportunity to review the commutative property of addition, and the definition of subtraction.

The Rectangle Find the areas of the rectangles whose widths are listed below. The rectangles all must have a perimeter of 30 inches. Therefore, the semi-perimeter ($L + W$) is 15 inches.

W in (L_1)	1	2	3	4	5	6	7	8	9	10	11	12	13	14
L in (L_2)														
A in (L_3)														



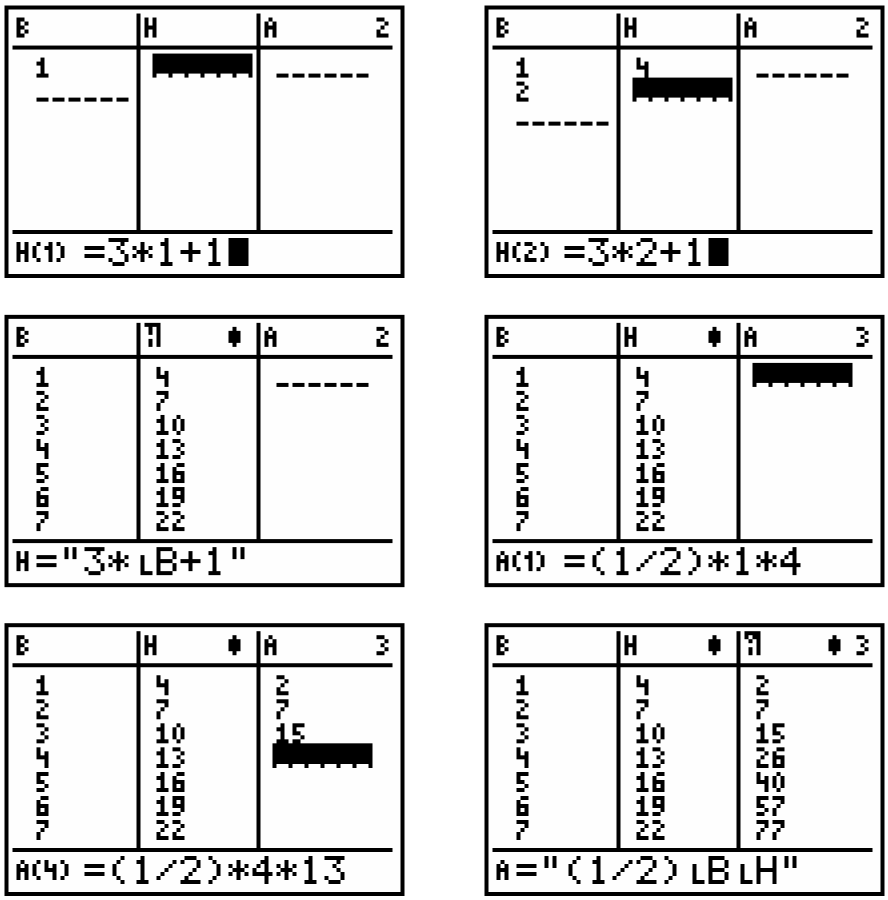




Here is a case where you really know the symbolic form of a quadratic function, but you may be thinking "formula" or "equation" and not function. If we could be so bold as to make a shift from L_1 to (x) and L_2 to ($15 - x$) notation, we see that the area is $x(15 - x)$, or $y = x(15 - x)$, or $y = -x^2 + 15x$.

The significant point is that by using an in-class discussion lesson plan or a student centered discovery activity, the student has a proof that data relationships lead to symbolic representation of the relationship, and that the symbolic form is just another representation of the data. Further, the symbolic representation can be used to generate the graphical representation of the data relationship. Make a graph of the data and the function. Make a table using the function.

The Artful Triangle For reasons unbeknownst to most, a modern artist wanted to create several triangles on her canvas whose height is <u>one more than 3 times the base</u>. In addition, she needs to know possible areas of these various triangles. Generate some data that leads to a model of the area of the triangles.



Here again is a case where you really know the symbolic form of a quadratic function, but you may be thinking "formula" and not function. Make a shift from B to (x) and H to ($3x + 1$) notation, we see that the area is $\frac{1}{2}x(3x + 1)$, or $y = \frac{3}{2}x^2 + \frac{1}{2}x$, or $y = 1.5x^2 + 0.5x$.

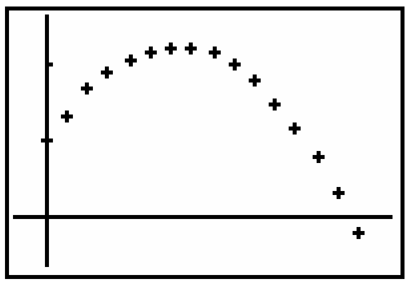
Be sure to graph (B, A) and $y = 1.5x^2 + 0.5x$ on a reasonable window, and follow this with exploring the numeric representation using TABLE.

The significant point is that by using an in-class discussion lesson plan or a student centered discovery activity, the student has a proof that data relationships lead to symbolic representation of the relationship, and that the symbolic form is just another representation of the data. Further, the symbolic representation can be used to generate the graphical representation of the data relationship.

The Ball Toss Below is the data relationship between time (in seconds) and the height (in feet from the floor/ground) of a ball as it is thrown straight upward. Time zero is the instant the ball leaves the hand throwing it. The data below is contained in the TI-83 Plus program TOMABALL.

time	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
height	5	6.8	8.4	9.6	10.4	11	11.2	11.2	10.8	10	9	7.6	6	4	1.6

If we look at the graphical representation, we get the impression that the data isn't linear, but more like a quadratic (or parabola).

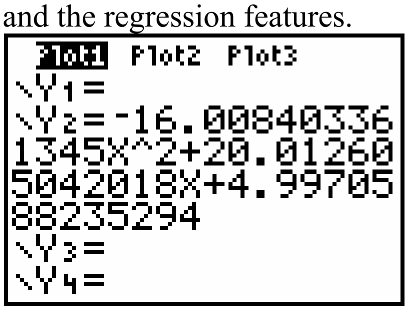


We know from the previous example the model should look something like $y = dx^2 + ex + f$. We have three options for finding the symbolic form of the data.

Option 1:
Many years ago, our physicist/mathematician colleagues figured out that the symbolic form of this kind of projectile motion is almost $y = -\frac{1}{2}gt^2 + v_0 t + y_0$, where g is the acceleration due to gravity, t is time in seconds, v_0 is the initial velocity, and y_0 is the initial height. So our first attempt might be $y = -16t^2 + v_0 t + 5$. We need to explore a little to find the initial velocity. What can we do?

Option 2:
Since we "know" that the symbolic form is something like $y = dx^2 + ex + f$, experiment a little with the parameters and a graph of the data. What do you think of this approach?

Option 3:
Use technology and the regression features.



I expect $y = -16x^2 + 20x + 5$ will be a good choice too.

Be sure to graph the data and $y = -16x^2 + 20x + 5$ on a reasonable window, and follow this with exploring the numeric representation using TABLE.

M & M's Are for More than Just Eating

Put students in groups of 2 – 3 each. Have each group record the number of M & M's in their packets under toss number 0. Then drop (toss) all M & M's on the table making certain they do not spread out too far or interfere with other groups. Count the number of M's, record this number as toss 1, and remove these from the activity. Collect and then drop (toss) the remaining M & M's. Count and record the number of M's and remove them from the collection. Continue using this process until you have no M & M's left to toss. If you need more than eight tosses, add them at the end of the current table. (Let the initial count at $t = 0$ be the number of M & M's in the packet.)

Toss (t)	0	1	2	3	4	5	6	7	8
# of M & M's (m)									

When all participating groups have finished, total the number of M & M's m from all groups for each toss t. This is the data you will be using for the remainder of the activity. Each person may enter the data in L_1 (t) and L_2 (m), or one person may enter the total data and then LINK it to all others in the groups.

Following the procedure below will lead to a model.

1. If you start with 50 M & M's, how many M's would you expect to show on the toss? Why?

____

____

2. If you had 7 non-M's on the fourth toss, how many M's would you expect on the fifth toss? Why?____

____

____

3. After studying the total data set, do you recognize any **numerical** pattern? If yes, describe the pattern in words. If no, explain what you were looking for that didn't materialize.

____

____

4. The parameter ½ is important in this activity. Why?_____________________________

____

5. Is the initial condition parameter (the number of M & M's the total group started with) important? How?____

____

____

6. **If** you graph the data, do you think it will be linear, quadratic, neither? Explain.__________

____

7. **Now**, graph the data. What name might you give the shape? What do mathematicians call it?

____

8. Using what you know from above answers, find a model for the data. You may also want to try mathematical reasoning and common sense. This may take a few educated guesses with refinements as you proceed to a final model. After making an attempt at a model, graph the model and the data. This should help you make a decision on how to improve the model. Hint: Use the standard form of a model.

1st Attempt________________Improvement to be made__________________________

2nd Attempt________________Improvement to be made__________________________

3rd Attempt________________Improvement to be made__________________________

NEED MORE? (use the other side)____

Final *M & M* model____

Teacher Note:
As another option for this activity, you may want to consider the use of the StudyCard App and the stack called (MANDMS.8xv). It is at a lower cognitive level because students are given 3 or 4 responses to each question above. Thus, there is no need for your students to create answers. The outcome is the same – symbolic form of the data.

TB Bacteria Below is the data relationship between time and the population of tuberculosis bacteria growing under unrestricted conditions. This is something like what might happen in a petri dish in a medical laboratory. Time is in hours and the bacteria population is in thousands.

Tuberculosis Data

t	0	6	12	18	24	30	36	42	48
B	5	10	20	40	80	160	320	640	1280

The initial conditions are shown and the bacteria population is doubling every 6 hours. The feature of "doubling" is represented symbolically as 2 raised to an exponent. What exponent? The student's first guess might be 2^0, but it produces a 1 and a 5 is needed. The second guess at symbolic form is usually 5×2^0, this will generate a 5 – shown in Figure 1. Thus 1 times 5 gives the correct value for the initial conditions.

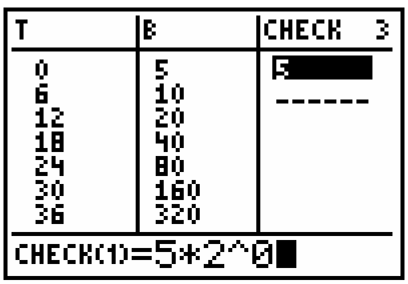


Figure 1

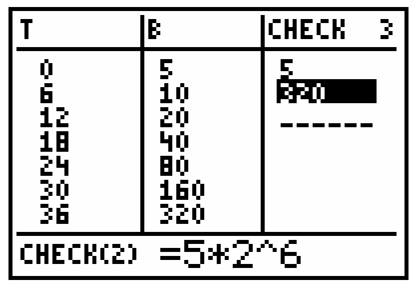


Figure 2

When *T* has a value of 6, students will guess 5×2^6, but it doesn't work for the number of bacteria at 6 hours – see Figure 2. The idea is OK but the exponent is too large. Likewise, 5×2^{12} doesn't work for the next data point. The *T* values are rising by 6, but the exponent needs to rise by 1. What arithmetic operation will take numbers that are increasing by 6 and make them

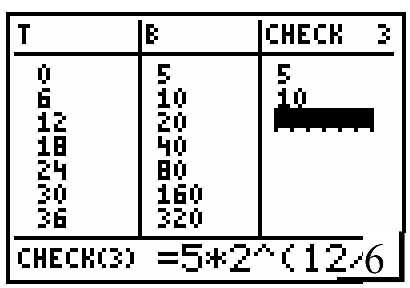


Figure 3

increase by 1? Division by 6. Figure 3 shows a correct guess.

Finally, the generalized symbolic form is $B = 5 \times 2^{\frac{T}{6}}$.

What we have accomplished is that our students now know that relationships existing in the real world can be represented symbolically. And that these "math" symbols aren't just stuff we make up. A low level of mastery is expected of students in being able to convert data or a situation into its' symbolic representation.

Summary

After completing the two sections above, you may want to consider using the Power Point presentation called ALGEBRA.PPT with your students. It was created by Clark Brown from Mohave Community College, and it is based on the presentation “Numeric, Graphic, and Finally Symbolic” made at the 1998 AMATYC Conference by Ed Laughbaum. Clark used most of the data relationships in the 1998 presentation (some of which are in this document), but added sound, motion, and clip art. It is available from Ed Laughbaum (with Clark’s permission).

Your students now think that all math symbols teachers and textbooks use are just symbolic representations of data relationships. At the remedial level, perhaps a case can be made for this. Given a set of symbols, we could probably find a real-world connection. However, you need to get to the point that at some level, mathematical symbols and the related manipulations do not have to relate to the real world. They may just be used to practice skill building of common mathematical procedures, or at other levels there may yet be no known real world connection.

Geometric Behaviors of Functions

(Solution to black box problem)

The last topic needed to finish the introduction to teaching remedial (developmental) algebra from a function approach is the understanding of function behaviors. Like in the previous two sections, do not expect complete mastery of the ideas presented. Full mastery should be gained when the ideas are taught in the context of complete chapters on the various functions.

Historically, we all know that a discussion of function behaviors was typically placed in Calculus I. When function graphers were developed in the mid to late 1980's, function behaviors were moved to pre-calculus textbooks and courses. When using a function approach to algebra, they can and must be included at this level. However, we do not expect the same level of rigor as you find in a calculus course. What we want and need is an intuitive understanding of them from our developmental (remedial) students, and in some cases, students must have the skills to find the behaviors. For example, students must be able to find the zeros of a function. They must be able to determine when a function is positive or negative. They must be able to find a maximum or minimum. They must be able to calculate an average rate of change. And they can do all of these tasks with the help of the graphing calculator, sometimes with pencil and paper, and sometimes they can be done mentally. The student activities that follow will prepare your students to do the mathematics.

In addition to using the graphing calculator as a teaching/learning tool, placing the study of function behaviors in the context of real-world situations makes this topic accessible to developmental students. Most of the following student activities are placed in a real-world context, and all behavior activities start in a context.

The topic of function behaviors is needed because without it there would be little understanding of later, more traditional, algebra topics. For example, teachers can "tell" their students that quadratic equations have none, one, or two real solutions. But telling does not often mean understanding. Likewise, "practicing" finding solutions to 40 or 50 quadratic equations also does not mean that students understand why there are none, one, or two real solutions, nor does it mean they understand what a zero means. But knowing the behaviors of the quadratic function like when positive, negative, or zero allows for understanding why this might be the case – none, one or two solutions to the related quadratic equations. Further, developing the behavior of "zero" in the context of real-world situations means that the odds of remedial students understanding what they mean goes up.

When learning about function behaviors, one might wonder if there is a connection between a function parameter (in symbolic form) and the related geometric function behavior? That is, for example, does the parameter d in $dx + e$ indicate anything about the behavior of the function? Likewise for the parameter e, what behavior does it control? What if students knew the behavior, could they find the parameter, and thus, the symbolic form of the function? As you can see, knowing the connections between function parameters and function behaviors is important. You will find student activities on this idea below.

Increasing/Decreasing/Maximum/Minimum

Micro Population

St. Maarten is a 37 square mile island situated in the Caribbean about 200 miles east of Puerto Rico. Philipsburg, St Maarten is the home of Great Bay Harbor and is the largest city in St. Maarten with a population of about 38,900 people. Great Bay Harbor is where cruise ships anchor to let passengers off to wonder the streets and shops of Philipsburg (mostly shops and restaurants). There are three ferryboats that move people from and to the cruise ships – La Nina, Pinta, and Santa Maria. Each ferry holds about 300 people. When there are more than two to three ships anchored in Great Bay, there are several smaller boats (Beachcomber I – XI) that can be used. Due to the limitations of the facilities and the customs procedures, it takes each boat 30 minutes to travel to the ship, load passengers, and return to the pier. When returning passengers go back to the ship, it takes a little longer since people return to the pier at various times and are in no great rush to return. The cruise ships usually arrive in early morning and anchor in Great Bay. The first ferryboat arrives at the dock at 9:00 am each morning. People are ferried in from 9:00 am until about noon. Starting about 1:30 PM, they start returning to the ship. The last ferry returns to the ship no later than 5:30 PM – it is usually 5:00 PM. During the off-season, usually one ship per day is in the harbor. During the tourist season, it is possible to have as many as seven cruise ships anchored in Great Bay Harbor. The normal is 4 or 5 ships per day.

Data was collected by the author during work on the beach and at Ric's Place, as he dined on curried goat. On April 10, 2000, the following data were collected. The data for other off-season days is very similar.

(STMAARTE)

Time	People from 1 cruise ship	Philipsburg population
9:00 am	300	39200
9:30	600	39500
10:00	900	39800
10:25	1200	40100
11:00	1500	40400
11:30	1500	40400
12:00 PM	1500	40400
12:30	1500	40400
1:00	1500	40400
1:30	1300	40200
2:00	1100	40000
3:10	900	39800
3:30	700	39600
4:00	500	39400
4:30	300	39200
5:00	100	39000
5:30	0	38900

Continued

1. When is the population of Philipsburg increasing? How do you know?___________________

____

2. If the Beachcomber ferries were used in addition to the La Nina, Pinta, and Santa Maria, would the population still be increasing during most of the morning hours? Explain.

____

3. When is the population of Philipsburg decreasing? How do you know?___________________

____

4. If the Beachcomber ferries were used in addition to the La Nina, Pinta, and Santa Maria, would the population still be decreasing during some of the afternoon hours? Explain.

____

5. When is the population of Philipsburg not increasing or decreasing? How do you know? How should we describe this situation of not increasing or decreasing?

____

____

6. What is the maximum number of people in Philipsburg on this day? When is it at maximum?

____

7. What is the minimum number of people in Philipsburg on this day? When is it at minimum?

____

8. On certain days, some people would not leave the cruise ship to go to Philipsburg, but rather, they would go snorkeling in the waters around St. Maarten. If this occurred in the data relationship on the previous page, would the maximum number of people in Philipsburg been affected? If so, how?

____

9. Would the minimum been affected? If so, how?

____

The Ball Toss

Below is the data relationship between time (in seconds) and the height (in feet from the floor/ground) of a ball as it is thrown straight upward. Time zero is the instant the ball leaves the hand throwing it. The data below is contained in the TI-83 Plus program TOMABALL.

Time	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
Height	5	6.8	8.4	9.6	10.4	11	11.2	11.2	10.8	10	9	7.6	6	4	1.6

1. When is the height of the ball increasing? How do you know?__________________________

   ____

2. If the ball had been thrown a little harder (larger initial velocity), how would your answer above be different?

   ____

   ____

3. When is the height of the ball decreasing? How do you know?__________________________

   ____

4. If the ball had been thrown a little harder (larger initial velocity), how would your answer in # 3 be different?

   ____

   ____

5. During the flight, when is the height of the ball not increasing or decreasing? Explain. How should we describe this situation?

   ____

   ____

6. At approximately what time does the ball reach its highest point?

   ____

7. What is the highest point the ball reaches?

   ____

Walking is Good for the Sole

Equipment needed: CBR, TI calculator, CBL 2 (optional, and nice)

Information: The CBR will collect time (in seconds) and distance from the CBR (in meters or feet) data for an object in the path of the CBR. Collection will begin at the push of the start key and run for a specified length of time.

Setup: From the DataMate app, select Setup to set Mode to Time Graph with 0.2 second between sample points for 15 or so samples. From the main menu, type 2 to collect data. The CBR will beep about **0.5 second or less** after you type 2 indicating the beginning of data collection. At this point, walk away from the CBR at a constant rate for 3 seconds. After viewing the time-distance graph, if you want to re-do the data collection go to the main menu and type 2 again. When you exit DataMate, please note where data is stored. After exiting, you are ready for the questions below.

<u>Collect data first. Use trace and look at TABLE to explore the data before proceeding.</u>

1. As you walked, was the distance from you to the CBR increasing?______________________

2. As you walked, was the distance from you to the CBR decreasing?______________________

3. Would the "time-distance relationship" be "increasing" if you walked toward the CBR?

   ____

4. How could you create a relationship that is decreasing?________________________________

5. What would the graph of the time-distance relationship look like if you varied your speed (rate of change)?

   ____

6. How can you create a "constant" time-distance relationship?

   ____

7. How can you create a time-distance relationship that is decreasing followed by increasing?

   ____

8. How can you create a time-distance relationship that has a maximum distance from the CBR?

   ____

Exploration Class________ Name________________________________

These data show the relationship between the Centigrade temperature and the weight of one gallon of water in pounds. (WTRWTDB)

Water Weight Data

T	0°	1°	2°	3°	4°	5°	6°	7°	8°
W	8.33461	8.33513	8.33551	8.33575	8.33585	8.33582	8.33565	8.33536	8.33495

Source: *CRC Handbook of Chemistry and Physics College Edition*. 1965-1966. The Chemical Rubber Company. Cleveland, OH

1. Describe the weight of one gallon of water around a temperature of 4˚ C.

____

A 12-ounce can of soda is equivalent to 354.8824 milliliters (*ml*). The volume of 354.8824 *ml* of water (soda is mostly water and will behave in a similar fashion) is measured at varying temperatures and the data is displayed in numeric form below. (POPCANDB)

POP Can Data

T	–10	–5	0	2	4	6	10	20
V	355.5425	355.1308	354.9285	354.8930	354.8824	354.8930	354.9782	355.5105

2. Describe the volume of the soda at a temperature of about 4˚ C.

____

Below is the numeric representation of the temperature *T* at time *t* in Columbus, Ohio during an "Alberta Clipper" cold front. (CLIPP124)

Clipper Data

t	0	1	2	3	4	5	6	7	8	9	10
T	2	0	–2	–4	–6	–8	–6	–4	–2	0	2

Time (*t*) zero is midnight.

3. Describe the temperature around 5 hours into the clipper.

____

Fill in the missing period data below by making a pendulum with the proper lengths (in feet).

Pendulum Data

l	0	½	1	1.5	2	2.5	3	3.5	4
t									

4. Describe the period as the arm length approaches 0 feet.

___

Below is the data relationship between time (in seconds) and the height (in feet from the floor/ground) of a ball as it is thrown straight upward. Time zero is the instant the ball leaves the hand throwing it. The data below is contained in the TI-83 Plus program TOMABALL.

Time	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
Height	5	6.8	8.4	9.6	10.4	11	11.2	11.2	10.8	10	9	7.6	6	4	1.6

5. Describe the height of the ball at around 0.6 seconds into the toss.

___

Conventional Mathematics (teacher activity)

Now that you have established an "existence" of a maximum and minimum, you can move to more conventional mathematics with your students like:

Find the maximum value of the function $y = -0.0000669\left(x-4.2\right)^2 + 8.33585$.
When is the function increasing? When is it decreasing?

Find the minimum value of the function $y = 0.00286x^2 - 0.0284x + 354.947$.
When is the function increasing? When is it decreasing?

Find the minimum value of the function $y = 2\left|x-5\right| - 8$.
When is the function increasing? When is it decreasing?

Find the minimum value of the function $y = 1.1107\sqrt{x}$.
When is the function increasing? When is it decreasing?

Find the maximum value of the function $y = -16\left(x-0.63\right)^2 + 11.4$.
When is the function increasing? When is it decreasing?

While asking the above questions of the function behaviors, also plot the data they model – see pages 39 and 40 above.

We do not necessarily want students to always "grab" the calculator when asked to find a minimum, maximum, when increasing or when decreasing. One solution is to emphasize the parameter-behavior connection. See next few pages and elsewhere in this document for activities on teaching this idea.

Parameter-Behavior Connection

It appears that this section (Parameter-Behavior Connection) has been inserted in the middle of the development of geometric behaviors. True, it has been because students have started picking up a sense that the function parameters are connected to function behaviors after studying increasing/decreasing and max/min. So, before continuing with the study of geometric behaviors of functions, we will pause and analyze the parameter-behavior connection.

We do not want students to always use a graph or table when asked to find a maximum or minimum, for example. We want them to be able to look at the symbolic form of a function and be able to identify what function parameter indicates a related behavior. So, starting on the next page you will find a series of discovery activities that will help your students recognize the connections.

Exploration Class________ Name________________________________

Find the indicated behavior for each function.

	When y changes from decreasing to increasing (or vice-versa)	Maximum or Minimum	Coordinates of the vertex
1. $y = 1(x + 3)^2 - 5$	____________	__________________	____________
2. $y = 2(x + 3)^2 - 5$	____________	__________________	____________
3. $y = 7(x + 3)^2 - 5$	____________	__________________	____________
4. $y = -1(x + 3)^2 - 5$	____________	__________________	____________
5. $y = -2(x + 3)^2 - 5$	____________	__________________	____________
6. $y = -7(x + 3)^2 - 5$	____________	__________________	____________

7. Does the function parameter d in the function $y = d\left(x+e\right)^2 + f$ control the coordinates of the vertex of the parabola? 7.________________

8. If $d > 0$ does the graph open up or down? 8.________________

9. If $d < 0$ does the graph open up or down? 9.________________

10. If $d = 0$ does the graph open up or down? 10.________________

11. What does parameter f control?____

____

12. What does parameter e control?____

____

13. Create a quadratic function that opens down and has a vertex at (–2, 5).

____

14. Create a quadratic function that has a vertex at (2, –5) and opens up.

____

Exploration Class________ Name__________________________________

Find the indicated behavior for each function.

	When y changes from decreasing to increasing (or vice-versa)	Maximum or Minimum	Coordinates of the vertex
1. $y = 1\|x+3\|-5$	____________	__________________	____________
2. $y = 2\|x+3\|-5$	____________	__________________	____________
3. $y = 7\|x+3\|-5$	____________	__________________	____________
4. $y = -1\|x+3\|-5$	____________	__________________	____________
5. $y = -2\|x+3\|-5$	____________	__________________	____________
6. $y = -7\|x+3\|-5$	____________	__________________	____________

7. Does the function parameter d in the function $y = d\left(x+e\right)^2 + f$ control the coordinates of the vertex of the parabola? 7.________________

8. If $d > 0$ does the graph open up or down? 8.________________

9. If $d < 0$ does the graph open up or down? 9.________________

10. If $d = 0$ does the graph open up or down? 10.________________

11. What does parameter f control?____

12. What does parameter e control?____

13. Create an absolute value function that opens down and has a vertex at (–2, 5).

____

14. Create an absolute value function that has a vertex at (2, –5) and opens up.

____

Concept Quiz Class________ Name________________________________

1. Create a quadratic function that has a minimum. 1.______________________________

2. Create a quadratic function that opens down and has a maximum value of 7. 2.______________________________

3. Create a quadratic function that opens up and has a minimum of 7. 3.______________________________

4. Create an absolute value function that has a minimum. 4.______________________________

5. Create an absolute value function that opens down and has a maximum value of 7. 5.______________________________

6. Create an absolute value function that opens up and has a minimum of 7. 6.______________________________

7. In the function $y = d\left|x+e\right| + f$, what are the coordinates of the vertex? 7.______________________________

8. In the function $y = d\left(x+e\right)^2 + f$, what are the coordinates of the vertex? 8.______________________________

9. Create a quadratic function that has a minimum of *a*. 9.______________________________

10. Create an absolute value function that has a maximum value of *a*. 10.______________________________

Class______________Hour________Name____

Transformation App Information:
The TI-83 Plus app used in this activity is called Transfrm on your calculator. To access it type the APP key and select Transfrm. Any function entered on the Y= editor may use the parameters A, B, C, and D in place of "normal" parameters used in your math textbook. You may then control these parameters using the WINDOW-SETTINGS editor.

For the exercises below, use a window like [–4.7, 4.7] × [–6, 6], Xres of 1, and app SETTING of >‖.

1. Using the linear function y = Ax + B, set the parameter A at 1 (under Window/Settings) and let the B parameter start at–5 with step of 1. Graph this function. You will see the function being graphed and current values of A and B. With a value for B of –5, trace to the intersection of the graph and the y-axis. What is the y-intercept? ______________________________

2. Type the GRAPH key on your calculator to bring back information on A and B. With the B=–5 highlighted, type the > key to increase B to –4. With a value for B of –4, trace to the intersection of the graph and the y-axis. What is the y-intercept? ______________________________

3. Type the GRAPH key on your calculator to bring back information on A and B. With the B=–4 highlighted, type the > key to increase B to –3. With a value for B of –3, trace to the intersection of the graph and the y-axis. What is the y-intercept? ______________________________

4. With the B=–3 highlighted, type the > key to increase B to –2. With a value for B of –2, trace to the intersection of the graph and the y-axis. What is the y-intercept? ______________________________

5. With the B=–2 highlighted, type the > key to increase B to –1. With a value for B of –1, trace to the intersection of the graph and the y-axis. What is the y-intercept? ______________________________

6. Continue increasing B in steps of 1 to 5. For each value of B, what is the y-intercept? ______________________________

7. For the linear function $y = x - 17$, what is the y-intercept? ______________________________
(No need for a graphing calculator.)

8. For the function $y = 2x - 17$, what is the y-intercept? ______________________________
(No need for a graphing calculator.)

9. For the function $y = -\frac{1}{2}x - 17$, what is the y-intercept? ____________________

10. Make a conjecture about what the y-intercept of function $y = \text{E}x + \text{F}$ is. ____________________

____

At this point, move back to the WINDOW SETTINGS editor and set B at 1 and let A be 0.5 with a step of 0.5. Graph the function.

11. With the A=0.5 highlighted, type the > key to increase A to 1. ____________________
With a value for A of 1, go to TABLE Set-up and start your table at 0 and let Δx (ΔTbl) = 1. Look at a table of this function and mentally, or with pencil and paper, find the change in x (Δx) from one point to another and find the corresponding change in y (Δy) from the above point to point. What is $\Delta y/\Delta x$ for this function?
[You may use any two points of your choosing.]

12. With the A=1 highlighted, type the > key to increase A to 1.5. ____________________
With a value for A of 1.5, go to TABLE Set-up and start your table at 0 and let Δx (ΔTbl) = 1. Look at a table of this function and mentally, or with pencil and paper, find the change in x (Δx) from one point to another and find the corresponding change in y (Δy) from the above point to point. What is $\Delta y/\Delta x$ for this function?
[You may use any two points of your choosing.]

13. With the A=1.5 highlighted, type the > key to increase A to 2. ____________________
With a value for A of 2, go to TABLE Set-up and start your table at 0 and let Δx (ΔTbl) = 1. Look at a table of this function and mentally, or with pencil and paper, find the change in x (Δx) from one point to another and find the corresponding change in y (Δy) from the above point to point. What is $\Delta y/\Delta x$ for this function?
[You may use any two points of your choosing.]

At this point, with A=2 highlighted, type the < key several times to change the value of A to −2.5.

14. With the A=−2.5 highlighted, type the > key to increase A to −2. ____________________
With a value for A of −2, go to TABLE Set-up and start your table at 0 and let Δx (ΔTbl) = 1. Look at a table of this function and mentally, or with pencil and paper, find the change in x (Δx) from one point to another and find the corresponding change in y (Δy) from the above point to point. What is $\Delta y/\Delta x$ for this function?
[You may use any two points of your choosing.]

15. With the A=–2 highlighted, type the > key to increase A to −1.5. ______________________________
With a value for A of −1.5, go to TABLE Set-up and start your table at 0 and let Δx (ΔTbl) = 1. Look at a table of this function and mentally, or with pencil and paper, find the change in x (Δx) from one point to another and find the corresponding change in y (Δy) from the above point to point. What is $\Delta y/\Delta x$ for this function?
[You may use any two points of your choosing.]

16. With the A=–1.5 highlighted, type the > key to increase A to −1. ______________________________
With a value for A of −1, go to TABLE Set-up and start your table at 0 and let Δx (ΔTbl) = 1. Look at a table of this function and mentally, or with pencil and paper, find the change in x (Δx) from one point to another and find the corresponding change in y (Δy) from the above point to point. What is $\Delta y/\Delta x$ for this function?
[You may use any two points of your choosing.]

17. With the A=–1 highlighted, type the > key several times to increase A to 0.With a value for A of 0, go to TABLE Set-up and start your table at 0 and let Δx (ΔTbl) = 1. Look at a table of this function and mentally, or with pencil and paper, find the change in x (Δx) from one point to another and find the corresponding change in y (Δy) from the above point to point. What is $\Delta y/\Delta x$ for this function? ______________________________
[You may use any two points of your choosing.]

18. Based on your observations from the last 7 exercises, make a conjecture on the value of $\Delta y/\Delta x$ for the function $y = 5x + 1$. ______________________________

19. Based on your observations from Exercises 11 – 17, make a conjecture on the value of $\Delta y/\Delta x$ for the function $y = -4x + 1$. ______________________________

20. Based on your observations from Exercises 11 – 17, make a conjecture on the value of $\Delta y/\Delta x$ for the function $y = -4x + 6$. ______________________________

21. Based on your observations from Exercises 11 – 17, make a conjecture on the value of $\Delta y/\Delta x$ for the function $y = \frac{1}{2}x - 3$. ______________________________

22. Based on your observations from Exercises 1 – 17, make a ______________________________
conjecture on the value of $\Delta y/\Delta x$ for the function $y = -\frac{2}{5}x + 12$, ______________________________
and find the y-intercept.

23. Based on your observations from Exercises 1 – 17, make a conjecture on the value of $\Delta y/\Delta x$ for the function $y = 4x + 12$, and find the y-intercept. ______________________________ ______________________________

24. The number $\Delta y/\Delta x$ is a measure of how fast the function is changing as x changes, or it is sometimes called the slope of the line. As slope, it measures how steep a graph is and whether it is increasing (rising) or decreasing (falling). Find the slope of the line $y = 3x + 2$ and list whether it is increasing or decreasing. ______________________________ ______________________________

25. Is $y = -4x + 7$ increasing or decreasing? What is its slope? ______________________________
____

26. Is $y = 3x - 2$ increasing or decreasing? What is its slope? ______________________________
____

27. Is $y = -5x - 3$ increasing or decreasing? What is its slope? ______________________________
____

Class______________ Hour________ Name_______________________________________

Transformation App Information:
The TI-83 Plus app used in this activity is called Transfrm on your calculator. To access it type the APP key and select Transfrm. Any function entered on the Y= editor may use the parameters A, B, C, and D in place of "normal" parameters used in your math textbook. You may then control these parameters using the WINDOW-SETTINGS editor.

For the exercises below, use a window like [–4.7, 4.7] × [–10, 10], Xres = 1, and app SETTING of >||.

1. Enter the quadratic function $y = A(x + B)^2 + C$ in the Y= editor. In the WINDOW-SETTINGS editor, set A to −8, B = 1, and C =1 with Step of 1. Graph the function and note whether it opens up or down. With A=–8 highlighted, type the > key to increase A by 1. Continue to increase A until it is −1. Does each graph open up or down while A < 0? ______________________

2. With A=–1 highlighted, type the > key to increase A by 1 to a zero value for A. Does the graph open up or down? ______________________

3. When A = 0, make a conjecture as to why the graph isn't a parabola. ______________________

   ____

   ____

4. With A=0 highlighted, type the > key to increase A to be a positive number. Note whether the graph opens up or down. Continue typing the > key. When A is positive, does the graph open up or down? ______________________

5. Without using your calculator, identify whether the following graphs open up or down.

   $y = 17(x + 1)^2 + 1$ ______________________

   $y = −17(x + 1)^2 + 1$ ______________________

   $y = 0.5(x + 1)^2 + 1$ ______________________

   $y = −(x + 1)^2 + 1$ ______________________

   $y = 2(x − 4)^2 − 3$ ______________________

6. On your graph screen, highlight the A= line. Using the < and > keys, investigate the following. List three functions (in symbolic form) that open up and are skinnier than $y = 2(x + 1)^2 + 1$.

7. On your graph screen, highlight the A= line. Using the < and > keys, investigate the following: List three functions (in symbolic form) that open down and are skinnier than $y = -5(x + 1)^2 + 1$.

8. Using the WINDOW SETTINGS editor, set A, B, and C to 1 with step of 1. Graph the function and then trace to the vertex of the parabola. What are the coordinates of the vertex? Highlight the B=1 line on the screen. Use the > key to increase B to 2. Trace to the vertex. What are the coordinates of the vertex? Repeat this process for B = 3 and 4.

9. With the B= line highlighted, use the < key several times to set B to 0. Use trace to find the vertex. What are the coordinates of the vertex? With the B= line highlighted, use the < key to set B to −1. Use trace to find the vertex. What are the coordinates of the vertex?

10. Repeat the above process for B = −2 and −3. What are the coordinates of the vertex at each value of B?

11. Using the WINDOW SETTINGS editor, set A, B, and C to 1 with step of 1. Graph the function and then trace to the vertex of the parabola. What are the coordinates of the vertex? Highlight the C=1 line on the screen. Use the > key to increase C to 2. Trace to the vertex. What are the coordinates of the vertex? Repeat this process for C = 3 and 4.

12. Without using your calculator, identify the vertex of each of the following parabolas determined by the quadratic functions.

$y = 1(x + 4)^2 + 3$ ____________________

$y = 1(x - 3)^2 - 2$ ____________________

$y = 3(x + 2.5)^2 + 1$ ____________________

$y = -2(x + 2)^2 + 3$ ____________________

$y = 2(x - 4)^2 - 3$ ____________________

13. Create quadratic functions in symbolic form that meet the conditions listed below: Please note that there are an infinite number of correct answers for each.

Opens up and has a vertex at (−1, 3) ____________________

Opens down and has a vertex at (2, −1) ____________________

Opens up, has a vertex at (0, 0) and is "skinnier" than $y = 5(x + 3)^2 - 7$. ____________________

Opens down, has a vertex at (1, −1) and is "fatter" than $y = 5(x + 3)^2 - 7$. ____________________

14. How do you know if $y = d(x + e)^2 + f$ opens up, down, or neither? ____________________

____

____

15. What are the coordinates of the vertex of the parabola determined by the graph of the quadratic function $y = d(x + e)^2 + f$? ____________________

16. What are the coordinates of the vertex of the parabola determined by the quadratic function $y = x^2 + 4x + 6$? ____________________

Class______________Hour________Name____

Transformation App Information:
The TI-83 Plus app used in this activity is called Transfrm on your calculator. To access it type the APP key and select Transfrm. Any function entered on the Y= editor may use the parameters A, B, C, and D in place of "normal" parameters used in your math textbook. You may then control these parameters using the WINDOW-SETTINGS editor.

For the exercises below, use a window like [–9.4, 9.4] × [–8, 8], Xres = 1, and app SETTING of >||.

1. Enter the exponential function $y = B^{(x + C)} + D$ in the Y= editor. ________________________
In the WINDOW-SETTINGS editor, set B = 2, C =0, and D = 1, with Step of 0.2. Graph the function and identify if the graph is increasing (rising to the right) or decreasing (falling to the right). With B=2 highlighted, use the > and < cursor keys to observe the graphs of several functions, while keeping B larger than 1. Are all the graphs increasing or decreasing?

2. With B= still highlighted, change B so that it takes on several ________________________
values between 0 and 1. Are all of these graphs increasing (rising to the right) or decreasing (falling to the right)?

3. Based on the above exercise, do you think the function ________________________
$y = 7^x + 1$ increasing or decreasing?

4. Based on the above exercise, do you think the function ________________________
$y = \left(\frac{1}{2}\right)^x + 1$ increasing or decreasing?

5. With B= still highlighted, change B so that it has a value of 1.________________________
Why doesn't the graph have the same shape as those above?

____

6. With B= still highlighted, change B so that it has a value of 0.________________________
Explain, as best you can, why you think the domain is all real numbers greater than 0. (The function is real when $x > 0$.)________________________

____

7. With B= still highlighted, change B so that it has <u>any</u> negative value. Propose a reason why mathematicians specify that B (the base) may not be negative or 0.

8. Do you think it is acceptable for B to have a value of 1? Explain.

9. Change B to 2 by using the < or > key. Or, go to WINDOW SETTINGS and change B to 2. Remember, C = 0 and D = 1 yet. Graph the function and use trace to find the *y*-intercept. What is it?

10. Using the above settings for C and D, change B to any number greater than 0. What is the *y*-intercept?

11. With B any number greater than 0, say 2.4. Highlight D= and change it to two. You may want to go to WINDOW SETTINGS and set Step to 1. What is the *y*-intercept?

12. With B any number greater than 0, say 2.4. Highlight D= and change it to three. What is the *y*-intercept?

13. Using the parameters $B \geq 0$, $C = 0$, and D is any number, what is the *y*-intercept? (You may express it symbolically.)

14. Change the parameter B to any number between 0 and 1, make $C = 0$, and D is any number, what is the *y*-intercept? (You may express it symbolically.)

15. In the WINDOW-SETTINGS editor, set $B = 2$ (or any number > 0 and not 1), $C = 0$, and $D = 0$, with Step of 1. Graph the function. With C=0 highlighted, use the < cursor key to observe the graphs of several functions. With C= highlighted, use the > cursor key to make $C > 0$, observe the graphs of several functions. Explain, as best you can, what you think the parameter C controls in the graph of the function $y = B^{(x + C)} + D$.

16. If you start with 10,000 M & M's, and toss them on a table, you expect about 5,000 to have the M facing up. If you remove the 5,000 (or so) with the M's facing up and toss the remaining 5,000 on a table, you expect about 2,500 to have the M's facing up. If you remove these and toss the remaining 1,250 on the table, you expect about, etc. A good model of the number of M & M's facing up on the x^{th} toss on the table is $y = 10000\left(\frac{1}{2}\right)^x$, where x represents the toss number, and y the number of M's facing up after the toss. Using the transfrm app, you can enter $y = 10000 \times 0.5$^x + D in Y1. In settings, set D = 0 and Step equal to 1. As the number of tosses increases, describe as best you can what you expect the number of M's facing up to approach? (Graph the function for future reference.)

____

17. What if we repeat the above activity, but this time after taking away all the M & M's with the M facing up, we add one M & M to the remaining pile. The model of this situation is $y = 1000\left(\frac{1}{2}\right)^x + 1$. (With D=1 highlighted, type the > key.) As the number of tosses increases, describe as best you can what you expect the number of M's facing up to approach?

____

18. What if we repeat the above activity (Exercise 16), but this time after taking away all the M & M's with the M facing up, we add two M & M's to the remaining pile. The model of this situation is $y = 10000\left(\frac{1}{2}\right)^x + 2$. (With D=1 highlighted, type the > key.) As the number of tosses increases, describe as best you can what you expect the number of M's facing up to approach?

____

19. What if we repeat the above activity (Exercise 16), but this time after taking away all the M & M's with the M facing up, we add three more M & M's to the remaining pile. The model of this situation is $y = 10000\left(\frac{1}{2}\right)^x + 3$. (With D=1 highlighted, type the > key.) As the number of tosses increases, describe as best you can what you expect the number of M's facing up to approach?

____

20. You should have observed that the graphs of the functions in the last four exercises approach a horizontal line as x increases. The line the graph is approaching is called a horizontal asymptote. Based on the activity in Exercises 16 – 19, find the horizontal asymptotes of the following exponential functions.

$y = 10\left(\frac{2}{3}\right)^x$ __________________________

$y = \left(\frac{1}{4}\right)^x + 5$ __________________________

$y = (3)^x - 7$ (This time, answer the question as x is decreasing.)__________________________

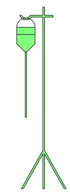
$y = (10)^x + 5$ __________________________

$y = -4(3)^x + 2$ __________________________

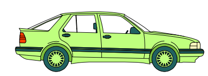
Function Behavior-Zeros

Exploration Class________ Name________________________________

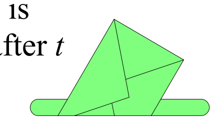
1. A 1000-*ml* I.V. is being administered to a hospital patient at a drip rate of 2.5 ml per minute. The function that models the amount of I.V. fluid left is $Amount = -2.5t + 1000$, where t is time in minutes. When will the I.V. bottle have no fluid left?



2. A small car (1988 Camry) with a 12.8-gallon gasoline tank averages 32 miles per gallon driven. The function that models the amount of gasoline left in the gas tank is $G = -\frac{m}{32} + 12.8$, where m is the number of miles driven. When will the gas tank be empty?

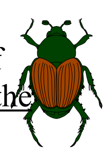


3. A postal worker has 3224 pieces of mail to sort before it can be delivered. He can sort at a rate of 1.2 pieces per second. The function that models the amount of mail left to sort is $m = -1.2t + 3224$, where t is time in seconds and m is the amount of mail left to sort after t seconds have passed. When is there no mail left to sort?



4. A window washer in the Dallas - Fort Worth Airport has 873 windows to wash before she can take a break. She can wash windows at a rate of 1 window every 12 seconds. The function that models the number of windows left to wash is $w = -\frac{t}{12} + 873$, where t is time in seconds. Why? When are there no windows left to wash?

5. On June 29, 1994, 15 Japanese beetles were sighted in Ed's red raspberry patch. Each day thereafter he observed 3 more beetles per day; thus, the function that models the number of beetles in the berry patch is $B = 3t + 15$, where t is in days. When were there no beetles in the berry patch?



6. If you throw a ball straight upward with an initial velocity of 6 feet per second and it leaves your hand when it is 5 feet above the ground, the model of the height of the ball is $h = -16t^2 + 6t + 5$, where t is measured in seconds. When will the ball have a zero height?

Exploration/Concept Quiz Class________ Name___________________________

1. Find the zero(s) of $y = x - 3$ 1. _______________________

2. Find the zero(s) of $y = x + 2$ 2. _______________________

3. Find the zero(s) of $y = (x - 3)(x + 2)$ 3. _______________________

4. Find the zeros(s) of $y = x - 7$ 4. _______________________

5. Find the zeros(s) of $y = x - 3$ 5. _______________________

6. Find the zeros(s) of $y = (x - 7)(x - 3)$ 6. _______________________

7. Find the zero(s) of $y = x + a$ 7. _______________________

8. Find the zero(s) of $y = x - b$ 8. _______________________

9. Find the zero(s) of $y = (x + a)(x - b)$ 9. _______________________

10. Create any function that has a zero of 8 10. _______________________

11. Create any function that has a zero of −5 11. _______________________

12. Create any function that has zeros of 8 & −5 12. _______________________

13. Create any function that has a zero of c 13. _______________________

14. Create any function that has zeros of d & c 14. _______________________

Concept Quiz Class________ Name________________________________

1. $y = |x-2| + 1$ has no zeros. Why? 1. ______________________

____

2. $y = |x-2| + 0.5$ has no zeros. Why? 2. ______________________

____

3. $y = |x-2| + 0.2$ has no zeros. Why? 3. ______________________

____

4. $y = |x-2| + 0.01$ has no zeros. Why? 4. ______________________

____

5. $y = |x-2|$ has a zero. Why? 5. ______________________

____

6. $y = (x-2)^2 + 1$ has no zeros. Why? 6.______________________

____

7. $y = (x-2)^2 + 0.5$ has no zeros. Why? 7.______________________

____

8. $y = (x-2)^2 + 0.1$ has no zeros. Why? 8.______________________

____

9. $y = (x-2)^2 + 0.01$ has no zeros. Why? 9.______________________

____

10. $y = (x-2)^2$ has a zero. Why? 10.______________________

____

Concept Quiz Class________ Name________________________________

1. Create a **linear** function that has a zero of 5. 1.________________________

2. Create a **linear** function that has a zero of −5. 2.________________________

3. Create a **linear** function that has no zero. 3.________________________

4. Create a **linear** function that has zero as the zero. 4.________________________

5. Create a **quadratic** function that has a zero of 5. 5.________________________

6. Create a **quadratic** function that has zeros of −5 and 5. 6.________________________

7. Create a **quadratic** function that has zeros of −2 and 2. 7.________________________

8. Create a **quadratic** function that has only one zero. 8.________________________

9. Create a **quadratic** function that has no zeros. 9.________________________

10. Create an **absolute value** function that has zero as a zero. 10.______________________

11. Create an **absolute value** function that has a zero of −2. 11.______________________

12. Create an **absolute value** function that has zeros of −2 and 2. 12.______________________

13. Create an **absolute value** function that has no zeros. 13.______________________

14. Create an **absolute value** function that has only one zero. 14.______________________

Functions that Are Positive and Negative

Exploration Class________ Name________________________________

Below is the numeric representation of the temperature *T* at time *t* in Columbus, Ohio as a quickly moving "Alberta Clipper" cold front goes through the city.

t	0	1	2	3	4	5	6	7	8	9	10
T	2	0	–2	–4	–6	–8	–6	–4	–2	0	2

Time (*t*) zero is midnight.

1. Describe when the temperature is a negative number.__________________________________

   ____

2. Describe when the temperature is a positive number.__________________________________

   ____

3. If the temperature is not positive or negative, what is it?______________________________

   ____

4. What do you think the temperature was at 1:30 am? Explain your reasoning.______________

   ____

   ____

5. What do you think the temperature was at 1:38 am? Explain your reasoning.______________

   ____

   ____

6. What do you think the temperature was at 8:45 am? Explain your reasoning.______________

   ____

   ____

7. What do you think the temperature was at 8:59 am? Explain your reasoning.______________

   ____

   ____

Exploration Class________ Name________________________________

Using interval notation (except for zeros), describe when each of the linear functions has a value less than zero, zero, and greater than zero. Approximate numbers are acceptable as long as you round to hundredths.

		$y < 0$	$y = 0$	$y > 0$
a.	$y = 4x + 7$			
b.	$y = -3x + 2$			
c.	$y = 0.06x - 9$			
d.	$y = -53x + 80$			
e.	$y = (\frac{1}{4})x + 2$			
f.	$y = -(\frac{3}{4})x + 5$			
g.	$y = \pi x + 1$			

Domain/Range

Exploration Class________ Name________________________________

A 1000-*ml* I.V. is being administered to a hospital patient at a drip rate of 2.5 ml per minute. The function that models the amount of I.V. fluid left is $Amount = -2.5t + 1000$, where *t* is time in minutes. In the model (function) $Amount = -2.5t + 1000$, is it mathematically acceptable for *t* to have a value of:

t	Yes/No	Why
−100		
−20		
−2		
−1		
0		
10		
100		
300		
400		
401		
410		
500		
1000		

In the model (function) $Amount = -2.5t + 1000$, given the context of the situation, is it acceptable for *t* to have values of the numbers below?

t	Yes/No	Why
−100		
−20		
−2		
−1		
0		
10		
100		
300		
400		
401		
410		
500		
1000		

Describe, as best you can, the differences in the two situations above.________________________

____

Exploration Class________ Name________________________________

Below is the data relationship between time (in seconds) and the height (in feet from the floor/ground) of a ball as it is thrown straight upward. Time zero is the instant the ball leaves the hand throwing it. (TOMABALL)

time	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
height	5	6.9	8.4	9.7	10.5	11.1	11.4	11.3	10.9	10.2	9.2	7.9	6.2	4.2	1.9

1. If the model of the data above is $y = -16(x - 0.63)^2 + 11.4$, is it mathematically acceptable for *x* (or *time*) to have a value of 1.5 seconds? Why?

____

____

2. If the model of the data above is $y = -16(x - 0.63)^2 + 11.4$, is it acceptable for *x* (or *time*) to have a value of 1.5 seconds – given the context of the situation? Why?

____

____

3. What is the minimum height of the ball – given the context? In the model, $y = -16(x - 0.63)^2 + 11.4$, what is the set of all *y* values (all heights) – given the context?

____

____

4. Ignoring the context, what is the set of all *y* values in the function $y = -16(x - 0.63)^2 + 11.4$?

____

____

5. What is the set of all *y* values in the function $y = 2(x - 1)^2 - 3$?________________________

____

Concept Quiz Class________ Name________________________________

Create an absolute value function that:

1. opens up and has a minimum of 3. 1.______________________________

2. opens down and has a maximum of 9. 2.______________________________

3. has a range of [–3, ∞). 3.______________________________

4. has a range of (–∞, 3]. 4.______________________________

5. has a domain of (–∞, ∞). 5.______________________________

Create a quadratic function that:

6. opens up and has a minimum of 3. 6.______________________________

7. opens down and has a maximum of 9. 7.______________________________

8. has a range of [–3, ∞). 8.______________________________

9. has a range of (–∞, 3]. 9.______________________________

10. has a domain of (–∞, ∞). 10.______________________________

Create a square root function that:

11. is increasing and has a minimum of 3. 11.______________________________

12. is decreasing and has a maximum of 9. 12.______________________________

13. has a range of [–3, ∞). 13.______________________________

14. has a range of (–∞, 3]. 14.______________________________

15. has a domain of [–3, ∞). 15.______________________________

16. What is the domain of the function $y = x + 1 - \sqrt{x+3}$? 16.______________________________

17. What is the domain of the function $y = x + 1 - 0\sqrt{x+3}$? 17.______________________

Teacher Notes on the Next Activity:

Instead of using the next activity, which requires that students create their own responses to the questions, you may choose to use the StudyCard App on the TI-83 Plus with the card stack “CHANGE.8xv”. It is identical to the following activity except for the fact that each item has a multiple-choice response. Thus, it is on a lower cognitive. At the same time, it is a little more fun than using pencil and paper. Also, using the study card stack means the activity should be worked by individual students and not in a group setting as you might assign with pencil and paper.

<u>*Average Rate of Change*</u>

(Over)

Exploration Class________ Name________________________________

1. A taxi company charges \$0.45 per ¼ mile, explain the meaning of "\$0.45 per ¼ mile."

____

2. A road sign near Naha City in Okinawa simply reads 5% grade. What is the meaning of this?

____

3. What is the meaning of "60 miles per hour?"

____

4. A hospital patient IV is set to release 20 drips per minute. What is the meaning of "20 drips per minute?"

____

5. A speed-reader can read 500 words per minute. What is the meaning of "500 words per minute?"

____

6. My real-estate property is taxed \$54 per \$1000 in assessed value. What is the meaning of "\$54 per \$1000?"

____

7. The economy is growing at a rate of 3% per year. Explain "3% per year."

____

8. I take 3 vitamin pills per day. What is the meaning of "3 pills per day?"

____

9. On a typical commercial airline flight, the plane ascends about 1200 feet per minute on its way to a cruising altitude. Explain what "1200 feet per minute" means.

____

10. The author of a textbook may earn $1.50 for every $10 in original sales of the book. What does "$1.50 for every $10" mean?

____

11. The roof on my house rises 12 feet for every 12 feet of horizontal width of the house. Explain "rises 12 feet for every 12 feet of horizontal width."

____

12. The bank charges me $0 per month for checking account services. What does "$0 per month" mean?

____

13. My printer can print 4 pages per minute. What does "4 pages per minute" mean?

____

14. My Internet browser can download information at about 2.5K per second. Explain "2.5K per second."

____

15. The earth is spinning on its axis and makes 1 revolution per day. What does "1 revolution per day" mean?

____

16. The US birth rate is 6850 people per day. What does "6850 people per day" mean?

____

17. The Japanese birth rate is 822 people per day. What does "822 people per day" mean?

____

18. A computer zip disk can hold 100,000,000 bytes of information per disk. What does "100,000,000 bytes per disk" mean?

____

19. **Describe one common feature found in all 18 exercises listed above.**

____

Exploration Class________ Name__________________________________

1. As x changes from 1 to 2 ($\Delta x = 1$) in the linear function $2x + 7$, how much does the function change [$\Delta(2x + 7)$]? 1.__________________

2. As x changes from 1 to 2 ($\Delta x = 1$) in the linear function $3x + 7$, how much does the function change [$\Delta(3x + 7)$]? 2.__________________

3. As x changes from 1 to 2 ($\Delta x = 1$) in the linear function $4x + 7$, how much does the function change [$\Delta(4x + 7)$]? 3.__________________

4. As x changes from 1 to 2 ($\Delta x = 1$) in the linear function $5x + 7$, how much does the function change [$\Delta(5x + 7)$]? 4.__________________

5. As x changes from 1 to 2 ($\Delta x = 1$) in the linear function $6x + 7$, how much does the function change [$\Delta(6x + 7)$]? 5.__________________

6. As x changes from 2 to 4 ($\Delta x = 2$) in the linear function $2x + 7$, how much does the function change [$\Delta(2x + 7)$]? 6.__________________

7. As x changes from 2 to 4 ($\Delta x = 2$) in the linear function $3x + 7$, how much does the function change [$\Delta(3x + 7)$]? 7.__________________

8. As x changes from 2 to 4 ($\Delta x = 2$) in the linear function $4x + 7$, how much does the function change [$\Delta(4x + 7)$]? 8.__________________

9. As x changes from 2 to 4 ($\Delta x = 2$) in the linear function $5x + 7$, how much does the function change [$\Delta(5x + 7)$]? 9.__________________

10. As x changes from 2 to 4 ($\Delta x = 2$) in the linear function $6x + 7$, how much does the function change [$\Delta(6x + 7)$]? 10.________________

11. Calculate the change in the function divided by the change in x, for Exercises 1 - 10. 11.____ ______

___ ______ ______ ______ ______ ______ ______

12. Describe the number: change in $dx + e$ divided by change in x? 12.________________

[$\Delta(dx + e) \div \Delta x$]